Walking-Induced Vibration of Composite Floor Systems

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ABSTRACT

The composite floor systems allow for long spans but can be susceptible to potentially objectionable levels of vibration. A dynamics model of a representative beam with elastic end conditions is developed and used to predict the resonance frequencies and mode shapes of a composite beam and the response of the beam to "walking" forces. The fundamental resonance frequencies for typical composite floor systems fall in the 4 to 5-Hz range, which is the frequency range people are also most motion-sensitive. Walking creates harmonics at multiples of about 1 Hz that can excite a floor system resonance and forced-response analyses indicate that walking can cause these systems to respond in the 0.002 to 0.02g range, which is perceptible, but may not be objectionable depending upon the intended use of the space. Variation in typical floor system design and the live load actually present accounts for system resonance frequency variability of about ±2 Hz, but it is virtually impossible to design a floor system with an adequate separation between the structure's resonance frequency and the excitation harmonics to completely avoid perceptible floor system motion.

INTRODUCTION

Composite floor systems are economical to construct and provide for a relatively column-free space in buildings. The yield stress of wide flange sections has increased to 50 ksi from 36 ksi over the past 15 years and widespread use of Load and Resistance Factor Design allow for longer spans. The relatively low stiffness-to-mass ratio of these structural systems imply greater susceptibility to human activity-induced motion. All floor systems are exposed to the repetitive application of forces as people walk across the floor. Repetitive forces are periodic and are characterized by a fundamental frequency and integer multiples thereof (the harmonics). There is the potential that one or more of the harmonics will coincide with a resonance frequency of the floor system giving rise to amplified motion. The response of typical modern-day composite floor systems to forces simulating walking is investigated to determine if the motions are sufficiently large to be considered objectionable.

Dynamics models representing an elastically-supported composite beam are developed and used to predict the resonance frequencies and mode shapes for several composite floor systems. A forcing function is defined and used in the dynamics model to predict the floor's motion caused by a single person walking across the floor. The predicted motion is compared to acceleration levels that humans can perceive and could find objectionable. The effect of floor system design parameters and their affect on the resonance frequencies is also studied to determine which structural design options may be most effective for mitigating excessive motion.

DERIVATION OF EQUATIONS

A plan view of the floor system of interest is shown in Figure 1. Purlins are spaced at a distance S and span a length L between the supporting columns or girders. Purlins are also assumed to be present on either side of the girders. The composite slab spans between the purlins, so the deck ribs are perpendicular to the purlin axes. The girders are also assumed to take advantage of composite action, but, in this case, the deck ribs are parallel to the beam's axis. While each of the purlins is structurally identical to its neighbors on either side, the location of the purlin along the girder affects its vibration characteristics. A purlin located at $0.5L_G$ ($s = 0.5$) will have a different resonance frequency than one located at $0.25L_G$ ($s = 0.25$), or one supported at a column ($s = 0$ or $s = 1$).

The model of the single purlin is shown in Figure 2. The beam has composite transformed geometric properties of area moment of inertia, I, and cross-sectional area, A. The mass density of the composite beam is ρ and any superimposed mass per unit length is μ. The modulus of elasticity is E.

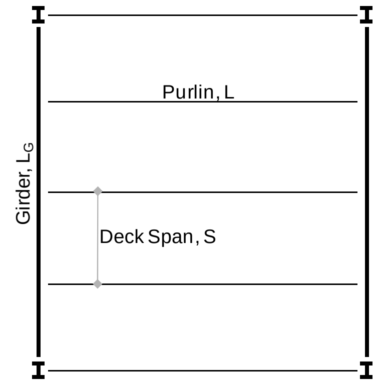

Figure 1 Composite Floor System

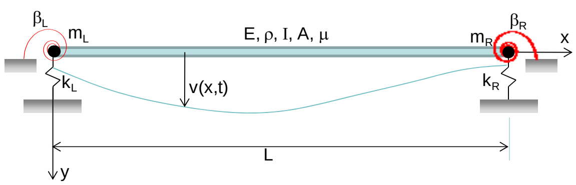

Figure 2 Elastically Supported Beam (Purlin)

Rotational springs with stiffness β_L and β_R are attached to the left and right ends of the beam, respectively. These represent the torsional stiffness of the girder (or the flexural stiffness of the column) that supports the beam and its connection (e.g., a double-angle connecting the web of the purlin to the web of the girder). These have zero stiffness for a simply supported beam or effectively infinite stiffness for a fixed-fixed beam. Zero stiffness is closer to the real-world application of interest here. Point masses m_L and m_R represent the tributary mass of an S-length section of the girder supporting the left and right ends of the beam. These point masses are not assumed to have rotational inertia because this effect is generally negligible.

Translational springs with stiffness k_L and k_R are also attached to the left and right ends of the beam. These represent the combined vertical stiffness from the supporting columns and the flexural stiffness of the supporting girder. Infinite stiffness reproduces a pinned support. The translational springs have the greatest influence on the purlin's vibration characteristics and can vary significantly because they account for the location of purlin along the girder. For any given purlin position sL_G $(0 \le s \le 1)$ from a supporting column, the translational spring stiffness is estimated from

$$k_L = k_R = \frac{1}{1/k_{Col,Left} + 1/k_{Col,Right} + 1/k_{Girder}} = \frac{EA_C}{L_C\left(1+2s^2-2s\right)+\left(1-s\right)^2 s^2 \dfrac{L_G^3 A_C}{3I_G}} \tag{1}$$

where A_C is the cross-sectional area of the column, L_C is its length, and the composite moment of inertia for the girder is I_G.

Hamilton's Principle is used to derive the governing equation of motion for a single elastically supported beam. This approach provides the governing partial differential equation for the vertical deflection $v(x,t)$ given definitions of the system kinetic energy, T, potential energy, V, and virtual work, δW, given by

$$\begin{aligned} T &= \frac{1}{2}m_L\dot{v}(0,t)^2 + \frac{1}{2}m_R\dot{v}(L,t)^2 + \int_0^L \frac{1}{2}(\rho A+\mu)\dot{v}(x,t)^2\,dx \\ V &= \frac{1}{2}k_L v(0,t)^2 + \frac{1}{2}\beta_L v'(0,t)^2 + \frac{1}{2}k_R v(L,t)^2 + \\ &\quad \frac{1}{2}\beta_R v'(L,t)^2 + \int_0^L \frac{1}{2}EIv''(x,t)^2\,dx \\ \delta W &= 0 \end{aligned} \tag{2}$$

where the primes denote differentiation with respect to x. The energy terms contain the typical beam assumptions that rotatory inertia is negligible and plane sections remain plane (i.e., essentially a large span-to-depth ratio). These assumptions lead to the standard Bernoulli-Euler equation for the beam. Carrying out the required substitutions and mathematical manipulation yields the dispersion relation, the relationship between the wavenumber, κ_n, and the vibration frequency, ω_n, given as

$$\kappa_n = \sqrt[4]{\frac{(\rho A+\mu)\omega_n^2}{EI}} \tag{3}$$

and the general solution for the deformed shape is

$$\begin{aligned} S(x) = a_n\sin(\kappa_n x) + b_n\cos(\kappa_n x) + \\ c_n\sinh(\kappa_n x) + d_n\cosh(\kappa_n x) \end{aligned} \tag{4}$$

The deformed shape is composed of transcendental functions rather than powers of x, which is the case for static deformation.

Hamilton's Principle produces the following partial differential equation and four boundary conditions for this model that are used to determine the unknown coefficients a_n, b_n, c_n, and d_n in Equation (4):

$$
\begin{aligned}
&EI\frac{\partial^4 v(x,t)}{\partial x^4}+(\rho A+\mu)\frac{\partial^2 v(x,t)}{\partial t^2}=0\\
&m_L\omega_n^2 S(0)-k_L S(0)-EIS'''(0)=0\\
&m_R\omega_n^2 S(L)-k_R S(L)+EIS'''(L)=0\\
&EIS''(0)-\beta_L S'(0)=0\\
&-EIS''(L)-\beta_R S'(L)=0
\end{aligned}
\tag{5}
$$

The boundary conditions are expressed in matrix form after substituting the general solution, Equation (4), and nondimensionalizing the terms to mitigate potential numerical solution problems

$$
\begin{bmatrix}
\alpha_n^3 & m_L'\alpha_n^4-k_L' & -\alpha_n^3 & m_L'\alpha_n^4-k_L' \\
\left[(m_R'\alpha_n^4-k_R')s(\alpha_n)-\alpha_n^3c(\alpha_n)\right] & \left[(m_R'\alpha_n^4-k_R')c(\alpha_n)+\alpha_n^3s(\alpha_n)\right] & \left[(m_R'\alpha_n^4-k_R')sh(\alpha_n)+\alpha_n^3ch(\alpha_n)\right] & \left[(m_R'\alpha_n^4-k_R')ch(\alpha_n)+\alpha_n^3sh(\alpha_n)\right] \\
-\beta_L'\alpha_n & -\alpha_n^2 & -\beta_L'\alpha_n & \alpha_n^2 \\
\left[\alpha_n^2s(\alpha_n)-\beta_R'\alpha_nc(\alpha_n)\right] & \left[\alpha_n^2c(\alpha_n)+\beta_R'\alpha_ns(\alpha_n)\right] & \left[-\alpha_n^2sh(\alpha_n)-\beta_R'\alpha_nch(\alpha_n)\right] & \left[-\alpha_n^2ch(\alpha_n)-\beta_R'\alpha_nsh(\alpha_n)\right]
\end{bmatrix}
\begin{Bmatrix} a_n \\ b_n \\ c_n \\ d_n \end{Bmatrix}
=
\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}
$$

$$
\alpha_n=\kappa_n L,\quad s(\alpha)=\sin(\alpha),\quad c(\alpha)=\cos(\alpha),\quad sh(\alpha)=\sinh(\alpha),\quad ch(\alpha)=\cosh(\alpha),\quad k_{L/R}'=\frac{k_{L/R}L^3}{EI},\quad m_{L/R}'=\frac{m_{L/R}}{(\rho A+\mu)L},\quad \beta_{L/R}'=\frac{\beta_{L/R}L}{EI}
\tag{6}
$$

The null right-hand side requires that the determinant of the 4×4 matrix be zero for a non-trivial solution to exist. An infinite number of values for α_n will drive the determinant to zero, and they must be found numerically for the general problem defined here. These are the eigenvalues for this matrix, which are used to obtained the resonance frequencies, ω_n, for the beam using Equation (3). The lowest values of α_n are of most interest because they are associated with the lowest resonance frequencies determined from

$$
f_n=\frac{\alpha_n^2}{2\pi L^2}\sqrt{\frac{EI}{(\rho A+\mu)}}\ \mathrm{Hz}
\tag{7}
$$

The corresponding mode shapes are found from Equation (6) using each distinct value of α_n and assuming, say, $a_n = 1$ and then solving for the remaining three coefficients. Mode shapes (i.e., the eigenvectors) have no absolute displacement—they provide only the relative displacement. The mode shapes are

$$
\begin{aligned}
\psi_n(x)=a_n\sin\left(\frac{\alpha_n x}{L}\right)+b_n\cos\left(\frac{\alpha_n x}{L}\right)+\\
c_n\sinh\left(\frac{\alpha_n x}{L}\right)+d_n\cosh\left(\frac{\alpha_n x}{L}\right)
\end{aligned}
\tag{8}
$$

RESONANCE FREQUENCIES OF REPRESENTATIVE FLOOR SYSTEMS

Representative composite floor system geometries based on Figure 1 are investigated in this section. All four structural systems employ a 3-in. deep metal deck and a 6-in thick normal weight concrete slab ($f_{c'}$ = 3 ksi). Post-composite floor loading is composed of a dead load of 58 psf (slab) and 10 psf (ceiling and mechanical) and a live load of 100 psf. SEI design procedures include an additional 19 psf in the dead load to account for possible excess concrete arising from the deflection of the non-composite beam during concrete placement. Construction (pre-composite) dead load is 58(+4) psf and the live load is 5(+15) psf. Floor system designs with and without the additional dead load are generated using the industry-standard design software RAM SBeam v3.0.

A cross-section of the floor system is shown in Figure 3. Any purlin in the cross-section is modeled by the equations derived in the preceding section. The values of k_L and k_R are altered based on the s-value for the purlin of interest using Equation (1). The most flexible system (lowest natural frequency) results when s = 0.5, which is at midspan of the girder. The current model does not account for coupled vibration of the purlin/girder system; instead, the girder stiffness is derived from its static deflection characteristics. The present model predicts the lowest resonance frequency of the purlin with reasonable accuracy.

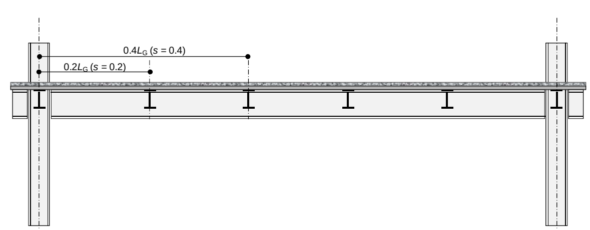

Figure 3 Cross-Section of Typical Floor System

Representative Composite Floor Systems

The primary span information for each of four representative systems is summarized in Table 1. Case 1 is arguably the most unrealistic because of the fairly short purlin span. This system is included primarily to illustrate the important role that the purlin length has on the resonance frequency. Cases 2 and 4 may be the most realistic. Cases 3 and 4 are the most flexible systems. The member design information is obtained using RAM SBeam. The SEI design procedures lead to different member sizes for Cases 1 and 3. Cases 2 and 4 require more shear studs for the SEI procedures, which slightly increases the effective composite moment of inertia.

Table 1 Representative Structural Floor Systems

Case	L	S	L_G	Member Selection
1	20	8	40	W12×14, W24×62 SEI: W12×14, W24×62
2	30	10	40	W16×26, W30×90 SEI: Same
3	40	8	40	W18×35, W30×116 SEI: W18×40, W33×118
4	40	10	40	W18×40, W33×118 SEI: Same

None of the live load is included as mass for the analyses discussed in this section. A fully loaded floor system (i.e., full live load) increases the mass and reduces the resonance frequency. Modes are determined for resonance frequencies up to 50 Hz. The mode shapes and resonance frequencies are provided in Figure 4 for Case 1 for two of the three unique purlin locations (s = 0.0 and 0.4). There are only two modes in the 0-50 Hz range when the purlin frames into the column (s = 0). The first and second modes have resonance frequencies of 8.1 and 32.2 Hz, respectively. These frequencies drop to 6.4 and 13.3 Hz, respectively, when the purlin frames into the girder at 0.4L_G. The girder flexibility reduces the resonance frequencies of the system even though the purlins are identical.

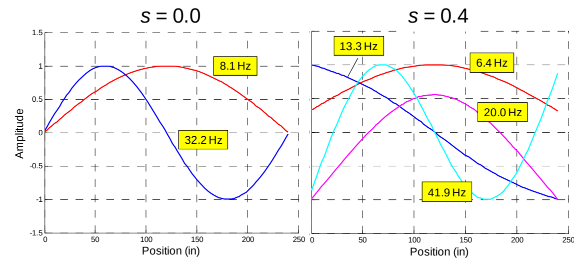

Figure 4 Resonance Frequencies and Mode Shapes for Case 1

The corresponding mode shapes provide valuable insight. For example, the first mode shape has a maximum at midspan, whereas the maximum amplitude for the second mode occurs at 0.24L and 0.76L[1] for s = 0. Also, the second mode shape passes through zero at midspan. This information implies that the first mode is most effectively excited when the source of excitation is at midspan. Likewise, the maximum response is also felt at midspan. Conversely, the same excitation cannot produce a response of the second mode because that mode shape is zero (has a "node") at midspan. The second mode is most effectively excited when the source acts at 0.24L (or 0.76L). An observer near either 0.24L or 0.76L would feel the greatest response at those locations, but another observer at 0.5L (at the node) would experience no vibration.

The frequency of the excitation is critically important as well. Most sources of real-world excitation are broadband; i.e., simultaneously excite a wide range of frequencies. Machinery with rotating components, however, can produce harmonic excitation at a single frequency. Other forms of excitation, like a person walking across the floor, produces periodic excitation with a fundamental frequency f_0 and harmonics at $2f_0$, $3f_0$, etc. The maximum response of any mode is obtained when the source of excitation is located where that mode shape has a maximum (positive or negative) and the excitation is harmonic with a frequency equal to that mode's resonance frequency. This combination drives the system into resonance and can damage the structure if the response amplitude is sufficiently large. Damping in the structure will ultimately limit

[1] The change in sign implies that as an observer at 0.24L is moving up, another observer at 0.76L is moving down at that instant.

the response amplitude, but the damage can occur before this limit is reached.

The location and frequency content of the excitation are both important. Consider, for example, a piece of rotating machinery with an excitation frequency of 32 Hz. Referring to Figure 4, one theoretically logical choice for locating the equipment on the floor is to place it at midspan of the purlin that frames into the column because that mode shape (which has a resonance frequency of 32.2 Hz) has a node at midspan, hence the equipment cannot provide any excitation to that mode even though the excitation frequency is very close to that mode's resonance frequency.

Cases 2, 3, and 4 are more realistic structural systems because they have longer purlin spans of 30 and 40 ft. The mode shapes and resonance frequencies are provided in Figure 5 for Case 2, in Figure 6 for Case 3, and in Figure 7 for Case 4.

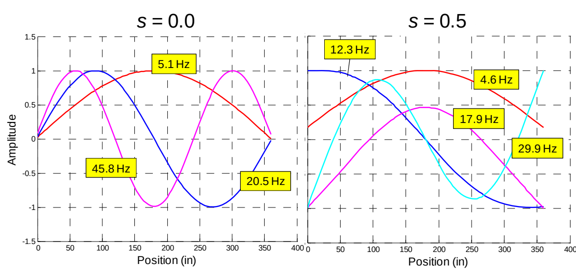

Figure 5 Resonance Frequencies and Mode Shapes for Case 2

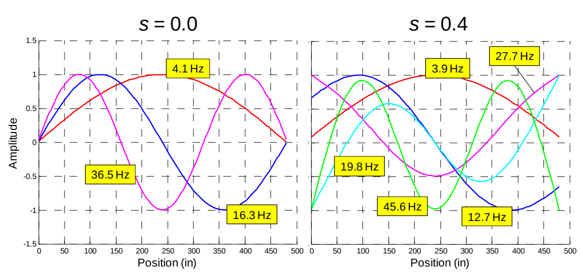

Figure 6 Resonance Frequencies and Mode Shapes for Case 3

These systems share qualitative similarities. The fundamental resonance frequencies fall in the 4- to 5-Hz band. The purlin at the column has the highest resonance frequency, while the purlin near the girder midspan has the lowest resonance frequency. There is a 0.5-Hz difference for Case 2, but only a 0.2-Hz difference for Cases 3 and 4. Lower overall system sensitivity to vibration results when the resonance frequencies are more distinct, because this condition helps to avoid all of the beams in a bay being excited at the same time by a single source of excitation. The fundamental resonance frequencies for each of the four structural systems studied here are summarized in **Error! Reference source not found.**.

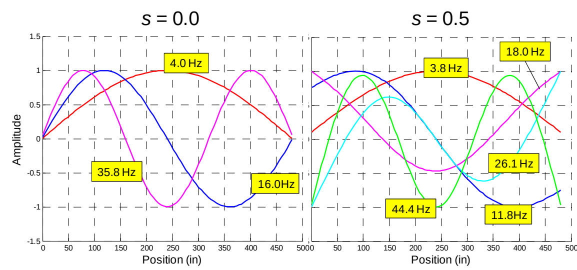

Figure 7 Resonance Frequencies and Mode Shapes for Case 4

Table 2 Summary of 1st-Mode Resonance Frequencies

Case	s = 0	s = 0.2,0.25	s = 0.4,0.5
1	8.1 Hz	7.3 Hz	6.4 Hz
2	5.1 Hz	4.8 Hz	4.6 Hz
3	4.1 Hz	4.0 Hz	3.9 Hz
4	4.0 Hz	3.9 Hz	3.8 Hz

All of the mode shapes have near-zero magnitudes near the support, when that support is a column. Hence, excitation acting near a column cannot cause vibration of the floor system. The purlin will vibrate when it is supported by the girder and the source of excitation acts near to or on the girder itself. The second mode shape, in particular, develops a maximum amplitude at the support as the purlin is moved closer to the girder midspan. This effect can be mitigated to some degree by employing a stiffer girder than required by strength considerations alone. While theoretically possible, it may not be practical to select a girder section stiff enough to appreciably influence the purlin mode shapes. Also, the more flexible the girder, the greater the separation between the purlin modes from the column to the girder midspan.

Effect of Design Parameters

A floor system with a very high fundamental resonance frequency is most desirable to avoid response from occupant activity; hence, the design parameters that affect the fundamental resonance frequency are of interest. The four structural system configurations studied here provide some insight into design parameters that affect floor system resonance frequencies and mode shapes. Purlin span length is the dominant parameter. This is also suggested by the form of Equation (7), where the resonance frequency is seen to be inversely proportional to the square of the span length. Doubling the span, reduces the resonance frequency by a factor of four. This effect is not as pronounced in practice, because a longer span requires a larger moment of inertia to satisfy strength requirements. In any event, it is seldom desirable or even possible to reduce the span length in buildings because it increases the construction cost and renders the floor space less usable because of the presence of more columns.

Given a bay size, it is possible to use longer-span purlins with a shorter-span girder or shorter-span purlins with a longer-span girder. This option only accounts for a ±1-Hz difference. The number of purlins can be increased, which reduces the superimposed mass on each purlin, thereby raising the resonance frequency. Considering Case 2, the W16×26 purlins are spaced at 10 ft, which produces a fundamental mode near 5.1 Hz. If the W16×26s are used at 5 ft on center instead, the resonance frequency increases to 7.3 Hz. This 2-Hz increase is not as significant as the doubling of the number of purlins is impractical.

A composite beam can be designed to provide full composite action or some percentage thereof. The American Institute for Steel Construction (AISC) [Reference (a)] requires at least 25% composite action, but SEI places the minimum requirement at 35%. Practical design scenarios often require partial composite action. For example, when the deck ribs are perpendicular to the beam—as they are in the case of the purlins here—the number of shear studs is limited by the number of ribs intersecting the beam. So there are practical limits to how many shear studs can be used. The composite moment of inertia depends on the degree of composite behavior as defined in Reference (a). When the beam does not act compositely with the slab (no shear studs), the moment of inertia is that of the steel beam alone. At the other extreme, when enough shear studs are used to develop the full yield strength of the steel beam and the slab is thick enough, the moment of inertia reaches its maximum value. The effect of composite action on the fundamental resonance frequency is shown in Figure 8 for Cases 2 and 4. The beams listed in Table 1 are designed for about 55% composite action, so the range of improvement spans from 55% to 100% in the figure. This range accounts for about a 0.5-Hz increase in the fundamental resonance frequency. This plot also suggests that merely using a stiffer beam section with a larger moment of inertia will not appreciably increase the resonance frequency, nor will the SEI design procedures have a significant effect.

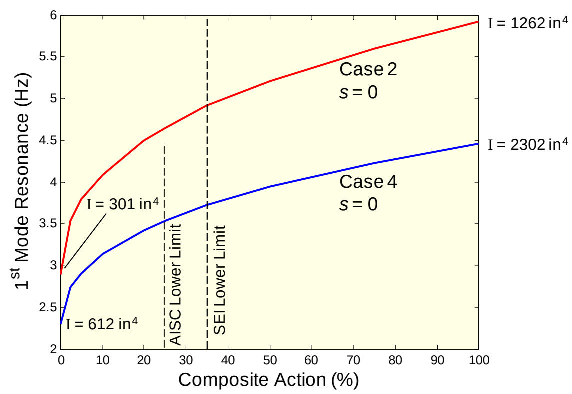

Figure 8 Effect of Number of Shear Studs on the Fundamental Resonance Frequency

Steel beam selection, bay definition, and the number of shear studs are under some degree of control by the engineer. The analyses presented here suggest it is very difficult to alter the resonance frequencies from the 4- to 5-Hz vicinity. The engineer has even less control over the superimposed mass and its effect on the resonance frequency. The slab and beam weight and the weight of mechanical systems and flooring represent the minimum mass of the system. The live load is also superimposed mass and can vary significantly during the building's use. Design live loads—100 psf for the cases studied here—are often much larger than the load the fully-loaded structure will actually see. The effect of the live load on the fundamental resonance frequencies is shown in Figure 9. The live load a floor system may actually experience falls in the 5 to 20% range, but the full range is considered in Figure 9. The analysis shows that the resonance frequency can decrease by about 1.5 Hz from an unloaded floor system to a fully-loaded floor system. This range of variation exceeds the effect of most of the structural parameters discussed above. Hence, any attempt to design a floor system to have (or avoid) a specific resonance frequency can be offset at various times during the day as the occupancy ebbs and flows.

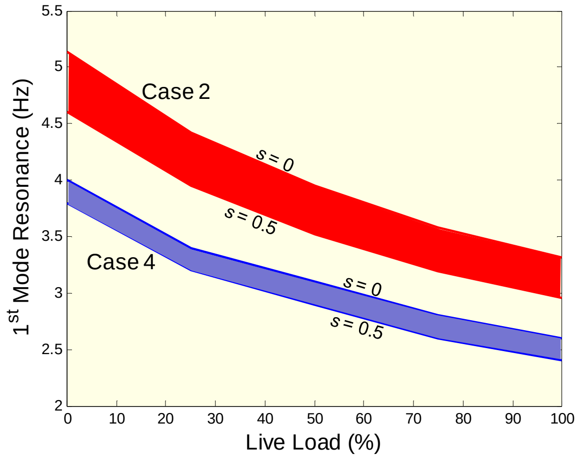

Figure 9 Effect of Live Load (Added Mass) on the Fundamental Resonance Frequency

FORCED RESPONSE ANALYSIS DUE TO PREDESTRAIN TRAFFIC

The resonance frequencies and mode shapes provide valuable insight into whether a given structural system may be susceptible to some form of excitation, but a modal analysis cannot provide a direct assessment of whether a specific form of excitation will tend to induce vibration that will prove disturbing to the occupants. A forced-response analysis is required to gain insight on this front.

Lagrange's Equations and the assumed modes method are used to develop a dynamics model for the floor system. It is also necessary to define a relevant forcing function. The effect of a person walking on the floor is studied here and a force-time history is defined to describe the force that a person's foot exerts on the floor during each step. This forcing function is used repetitively to simulate walking. The successive footfalls can be applied at different locations along the purlin length to simulate the effect of a person walking in the direction of the purlin's span. Given the forcing function, the acceleration response can be determined at any location on the purlin. Acceleration is the most relevant metric because people are sensitive to acceleration rather than to displacement or velocity. The predicted acceleration is then compared to acceleration levels that are considered to be perceptible and/or objectionable.

Derivation of the Equations of Motion

A multi-degree-of-freedom (MDOF) dynamics model is developed to predict the vibration response of the purlin to a known forcing function. The excitation considered here is intended to be representative of forces applied while walking. The step forces are modeled as a sequence of downward-acting forces applied at successive increments of time and, if desired, at successive increments of distance along the beam. The mass of the person is ignored because it is negligible compared to the other mass. The beam is shown in Figure 10. The assumed modes method is used in conjunction with Lagrange's Equations to derive the MDOF model.

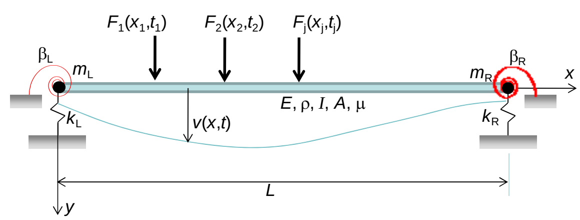

Figure 10 Forced Response Model

The assumed modes method discretizes the response, $v(x,t)$, into a sum of N assumed modes and employs the separation of variables concept. Hence,

$$v(x,t) \approx \sum_{n=1}^{N} q_n(t)\psi_n(x) \tag{9}$$

where the $\psi_n(x)$ are the mode shapes obtained as discussed in the previous section and defined in Equation (8). The unknowns in this analysis are the time-dependent amplitude functions, the $q_n(t)$. Lagrange's Equations act on the system kinetic and potential energy and virtual work and yield $N \times N$ matrices for the mass and stiffness and an $N \times 1$ load vector.

$$T = \frac{1}{2} m_L \left(\sum_{n=1}^{N} \dot{q}_n \psi_n(0) \right)^2 + \frac{1}{2} m_R \left(\sum_{n=1}^{N} \dot{q}_n \psi_n(L) \right)^2 + \int_0^L \frac{1}{2} (\rho A + \mu) \left(\sum_{n=1}^{N} \dot{q}_n \psi_n(x) \right)^2 dx \quad (10)$$

$$V = \frac{1}{2} k_L \left(\sum_{n=1}^{N} q_n \psi_n(0) \right)^2 + \frac{1}{2} k_R \left(\sum_{n=1}^{N} q_n \psi_n(L) \right)^2 + \frac{1}{2} \beta_L \left(\sum_{n=1}^{N} q_n \psi'_n(0) \right)^2 + \frac{1}{2} \beta_R \left(\sum_{n=1}^{N} q_n \psi'_n(L) \right)^2 + \int_0^L \frac{1}{2} EI \left(\sum_{n=1}^{N} q_n \psi''_n(x) \right)^2 dx$$

$$\delta W = F_1(x_1, t) \left(\sum_{n=1}^{N} \delta q_n \psi_n(x_1) \right) + F_2(x_2, t) \left(\sum_{n=1}^{N} \delta q_n \psi_n(x_2) \right) + \ldots + F_m(x_{m1}, t) \left(\sum_{n=1}^{N} \delta q_n \psi_n(x_m) \right)$$

A set of N coupled second-order ordinary differential equations (ODEs) is obtained and conveniently expressed in matrix form. The elements of the i^{th} row and j^{th} column of the mass matrix, **[M]**, the stiffness matrix, **[K]**, and the i^{th} element of the load vector **{F(t)}** are defined below:

$$[\mathbf{M}]\{\ddot{q}\} + [\mathbf{K}]\{q\} = \{\mathbf{F}(t)\}$$

$$\mathbf{M}_{i,j} = m_L \psi_i(0) \psi_j(0) + m_R \psi_i(L) \psi_j(L) + \int_0^L (\rho A + \mu) \psi_i(x) \psi_j(x) dx \quad (11)$$

$$\mathbf{K}_{i,j} = k_L \psi_i(0) \psi_j(0) + k_R \psi_i(L) \psi_j(L) + \beta_L \psi'_i(0) \psi'_j(0) + \beta_R \psi'_i(L) \psi'_j(L) + \int_0^L EI \psi''_i(x) \psi''_j(x) dx$$

$$\mathbf{F}_i(t) = \sum_{j=1}^{m} F_j(x_j, t) \psi_i(x_j)$$

The mass and stiffness matrices are symmetric which allows application of the modal superposition technique to solve for the response of each mode. Modal damping is introduced to account for energy loss mechanisms that exist in all structures. A relatively low level of damping (2% of the critical value) is used for all of the modes in the MDOF model. The number of modes used in the solution is dictated by an upper bound on the resonance frequency determined from the frequency content in the excitation. An upper bound of 100 Hz is used here.

Excitation: The "Walking" Function

Floor systems in buildings are subjected to a variety of time-dependent forces. Mechanical systems often have rotating elements (fans, turbines, etc.), that can produce a source of harmonic (single frequency) excitation. Mechanical systems can directly excite a floor vibration mode, so these systems deserve special consideration. A more common source of excitation derives from the building's occupants as they walk through the space. Walking produces periodic excitation in a vertical direction and in the plane of the floor, normal to the direction the person is walking. The lateral periodic excitation resulting from a large group of people was responsible for exciting a lateral mode of London's Millennium Bridge [Reference (b)]. Lateral excitation is not a concern for the floor systems studied here; however, the vertical excitation can produce undesirable vibration and is of interest.

The floor structure will deflect and dynamically respond (i.e., vibrate) after each step; and other occupants on that floor can sense the resulting vibration. More often than not, the motion is either not felt or not sufficiently large to cause discomfort. People are most sensitive to vibration in the 4- to 8-Hz band [Reference (c)], which coincides with the first mode resonance frequencies of the composite floor systems discussed above. Hence, forms of excitation that can excite the fundamental mode of the floor system can lead to occupant discomfort.

Walking produces a periodic form of excitation because the time between each step occurs at a regular interval. The time between each step, Δt, is about 1 sec; hence, a downward force is applied to the floor at 1-sec intervals. The magnitude of the force applied to the floor varies with time. The heel contacts the floor initially and the force increases rapidly as the area of contact between the foot and the floor increases. The force reaches a maximum and then decrease as weight shifts to the ball of the foot and the other heel contacts the floor. A representative force time history for five steps is plotted in Figure 11. The inset plot in the figure is taken from Reference (c) where an actual measured "step" force is provided.

The measured force shows a rapid rise to 600 lbf, followed by a more gradual decrease in the force to 0 lbf. Reference (c) suggests a triangular idealization (the dashed line in the inset plot) of this force history. These findings are used as a guide for this study. A peak force of 400 lbf is used instead of 600 lbf. The force-time history is modeled here as a quarter-sine wave during the initial rise and a half-cosine wave for the decreasing-force phase. Each footfall is identical, but shifted in time by Δt and stays in contact with the ground for a time period t_d. The overlap between successive steps accounts for the time when both feet are on the ground simultaneously (unlike running, where only one foot is in contact with the ground at any given time). The total force applied to the floor is the sum of the individual footfalls, although the forces may be applied at different locations on the floor.

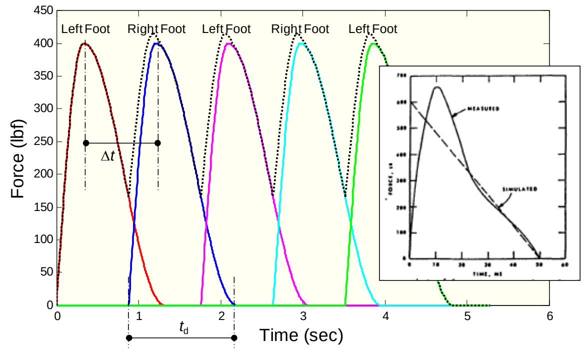

Figure 11 Assumed Force Time History for Successive Steps

A 10-step force-time history is plotted in the left-side plot in Figure 12 representing the total force (left foot plus right foot) versus time. A 0.9-sec interval is used for Δt, so the fundamental frequency of excitation is $f_0 = 1/\Delta t = 1.11$ Hz. The time history on the left-hand side is transformed into the frequency domain using the Fourier Transform and plotted in the right-hand side of Figure 12.

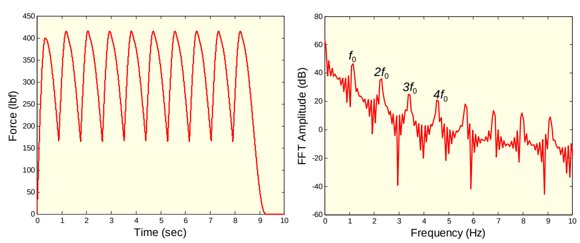

Figure 12 10-Step Force-Time History and Corresponding Frequency Domain

Peaks in the frequency domain correspond to the fundamental frequency and its harmonics at $2f_0$, $3f_0$, etc. The level of excitation decreases with each higher harmonic[2]. The third, fourth, and fourth harmonics ($3f_0$, $4f_0$ and $5f_0$) fall in the frequency range of the composite floor system fundamental resonance frequencies and can therefore excite that resonant mode depending upon the actual Δt and the actual fundamental resonance frequency of the floor system.

Floor System Response

The 10-step force-time history defined above is used as the forcing function for the MDOF model derived earlier. The fourth-order Runge-Kutta algorithm is used to solve the ODEs. The analyses presented here focus on the structural systems in Cases 2 and 4 and assume that the person is walking perpendicular to the purlin at $x = 0.35L$ (not quite at midspan). The acceleration response that another person would experience is obtained at midspan (0.5L) and near the support at 0.1L. The level of vibration considered to be objectionable is subject to interpretation. Reference (c) suggests 0.4g for offices and "rhythmic activities," while Reference (d) offers 0.005g as a threshold. The duration of the vibration is also a factor. The longer the vibration persists, the lower the threshold that is tolerable. For the purposes of this study, the 0.005g-limit is adopted as the "perceived" limit, with the interpretation that the vibration could be problematic for the occupants. Of course, some intended uses such as laboratories or sensitive component manufacturing spaces require stricter limits.

The structural systems represented by Cases 2 and 4 are used because they are the most realistic with purlin spans of 30 or 40 ft. Case 4 is the more flexible of the two systems with a fundamental resonance frequency of 3.8 Hz compared to the Case 2 system, which has a 4.6-Hz resonance frequency. The acceleration responses of the two identically-excited floor systems are plotted in Figure 13 for the purlins located at the girder midspan. The 0.005g limit is shown as two horizontal dashed lines. The red and blue lines correspond to the responses at 0.5L and 0.1L, respectively. The highest response occurs at midspan, as suggested by the mode shape for the first mode. The responses for both systems exceed the "perceptible" limit of 0.005g, but the stiffer 30-ft purlin (Case 2) response is three times higher than the more flexible 40-ft purlin (Case 4) and is perceptible by others on the floor, whereas the response for Case 4 is not likely to be felt by other occupants. This result appears somewhat counterintuitive, but the frequency domain provides valuable insight.

[2] A factor of 2 reduction is equivalent to a 6 dB decrease.

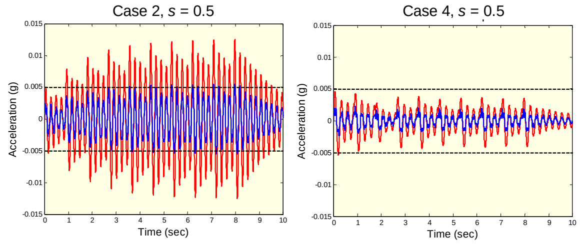

Figure 13 Floor Acceleration at 0.5L (Red) and 0.1L (Blue) for Cases 2 and 4

The frequency domain of the 10-step force time history is repeated in Figure 14. The fundamental resonance frequency of 4.6 Hz for Case 2 is shown as a line in the plot on the left-hand side of the figure. Similarly, a line representing the 3.8-Hz resonance frequency for Case 4 is shown in the right-hand plot. The critical difference is that the floor system resonance falls on top of the fourth "walking" harmonic for Case 2, whereas the Case 4 system resonance falls in the trough between the third and fourth harmonics. In other words, the fourth harmonic in the forcing function directly excites the Case 2 floor system resonance, which magnifies its response as compared to the more flexible Case 4 system.

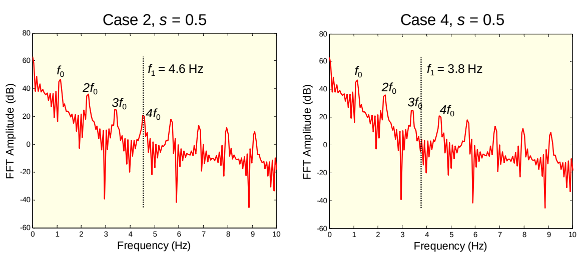

Figure 14 Walking Function Harmonics in Relation to Fundamental System Resonance

A 40-ft span is not necessarily preferable to a 30-ft span however. Recall Figure 9 and note that if 15% of the total live load is present (very reasonable), the roles would be reversed. In this case, the fundamental resonance frequency for Case 4 is 3.4 Hz, which coincides with the frequency of the third harmonic in the forcing function. Meanwhile, the fundamental resonance frequency for Case 2 falls to 4.1 Hz, which is between the third and fourth harmonics; hence, the first mode for Case 2 is not now directly excited.

Another example is shown in **Error! Reference source not found.**, where the response of the Case 2 system is plotted for the purlin that frames into the column ($s = 0$) instead of the girder midspan ($s = 0.5$). In this case, the resonance frequency is 5.1 Hz instead of 4.6 Hz because the girder flexibility is eliminated from the system. The fourth harmonic no longer excites a floor system resonance (see the frequency domain in the right-hand side plot). In this case, the floor system peak acceleration drops by a factor of five and probably cannot be felt by others nearby.

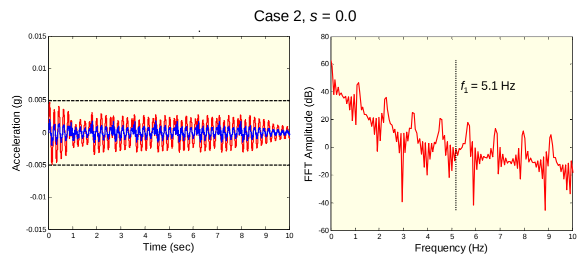

Figure 15 Floor Acceleration Response at 0.5L (Red) and 0.1L (Blue) for Case 2, *s* = 0

The walking force-time history is considered to be a "given" in this analysis. In fact, different people walk at different rates. Indeed, the same person may walk faster at some times and slower at other times. A person walking more slowly (slightly larger Δt) would shift the harmonics to lower frequencies. Conversely, walking faster shifts the harmonics to higher frequencies and probably results in a higher peak force for each step.

CONCLUSIONS

Four representative composite floor systems with purlin spans ranging from 20 ft to 40 ft are considered for this study. A single-purlin elastic vibration mathematical model is developed and used to predict the resonance frequencies and mode shapes of the purlins in each system. Except for the less reasonable 20-ft span system, the composite floor systems have fundamental resonance frequencies in the 4- to 5-Hz range, which coincides with the frequency band in which people are most sensitive to vibration. The most significant vibration response occurs when a fundamental mode is directly excited. Harmonic excitation provides the greatest risk, but typical cyclic machinery operation (e.g., fan revolution) tends to occur at higher frequencies. Rhythmic human activities such as aerobics, jogging, dancing, and walking have fundamental frequencies in the 1- to 4-Hz range. Of these, walking is obviously the most common and is studied here.

A periodic forcing function representing the vertical forces resulting from a person walking across the floor is developed and shown to have a fundamental frequency near 1 Hz and harmonics at integer multiples of the fundamental frequency. The force level at the fundamental is largest and then decreases with each successive harmonic. The third and fourth harmonics have the greatest chance to excite a floor system resonance given typical construction practices, but the force levels for those harmonics are between 20 to 25 dB lower (i.e., 10 to 18 times lower) than the force level at the fundamental frequency which mitigates the response at resonance.

An MDOF forced-response dynamics model is developed to predict the floor system acceleration caused by a 10-step idealized footfall force-time history. Peak responses between 0.002 to 0.02g are obtained depending upon whether an excitation harmonic coincides with the floor system resonance frequency. Motion exceeding 0.005g is considered to be perceptible but not necessarily objectionable. The intended use of the space also has a significant impact on whether the occupants would find the predicted vibration levels troublesome. The predicted motion may be objectionable if the subject area is exposed to high traffic throughout the day while others are working at their desks nearby. Less persistent motion may be objectionable if the space is dedicated to high-precision work or manufacturing where small levels of motion could impair the work.

Assuming the Architect/Engineer design team considers the subject space and intended use to be motion sensitive, the question arises as to what structural options can most successfully be employed to avoid potential problems. The ideal solution is to design a floor system with a high fundamental resonance frequency, say, greater than 8 Hz. The analyses performed here show that simply using stiffer structural members produces only a 1-Hz increase in the fundamental resonance frequency. A 1-Hz change in the resonance frequency can also result from variation in the live load—the resonance frequency for a floor system will be about 1-Hz higher when the system is lightly loaded (lower superimposed mass) than when a significant percentage of the rated live load is present. Purlin span length has the greatest effect, but structural bays must be on the order of 20×20 ft for typical simply-supported girders and purlins to achieve resonance frequencies of about 8 Hz. If fewer columns are desired for cost or architectural reasons, a change in the structural system may be required (e.g., a reinforced concrete frame or designing connections for moment continuity).

The long span composite floor system is desirable because of its relatively low erection cost and minimal floor space intrusion by supporting columns. A multistory office building may contain only a few areas where motion sensitivity is a concern. In these cases, it does not make sense to adopt a more costly structural system to mitigate the vibration in only a few locations. Vibration control options such as tuned-mass dampers and vibration isolators may be successfully applied in some cases. The potential effectiveness and design considerations associated with these options are investigated in a separate report.

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