# Using a Simple Cantilever Tuned-Mass Damper to Control Vibration in a Preexisting Composite Floor System

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## ABSTRACT

*The vast majority of composite floor systems exhibit no vibration problems, but unusual situations do arise where the structural configuration and/or occupant activities render a space more susceptible to vibration and require some form of post-construction mitigation. A practical low-profile tuned-mass damper (TMD) is designed to mitigate the walking-induced vibration of an already-existing composite floor. The degree of vibration mitigation is strongly affected by the added TMD mass. The TMD designed for this application is a cantilevered plate with a 200-lbf tip mass. Two TMDs are bolted to the bottom flange of the steel beam at midspan and require only 1 in. of clearance below the beam. The TMDs effectively reduce the acceleration in the motion-sensitive frequency band by a factor of 17.*

## INTRODUCTION

Composite floor systems in modern buildings have fundamental resonance frequencies in the 4- to 5-Hz range, which is a frequency band in which people are most sensitive to motion. Occupant discomfort caused by floor vibration is rare but does occasionally occur. A common source of vibration is produced by people as they walk across the floor. Harmonics in this periodic form of excitation can excite a floor system resonance causing low-level vibration that could be potentially disturbing, depending upon how the space is used. These problems are frequently identified only after construction is complete and the structure is occupied. Methods of providing vibration mitigation that can be implemented with minimal interference with the existing structure and the building's occupants are of interest.

A tuned-mass damper (TMD) provides one means of passive vibration control (i.e., no external source of power) and its application for retrofitting an existing floor system is illustrated in this paper. In its simplest form, a TMD consists of a spring and a mass, and the ratio of the two is selected (i.e., tuned) to a specific frequency. In most applications, the TMD frequency is tuned to the frequency of excitation, which often coincides with a resonance frequency of the structure. The TMD's motion is out of phase with the excitation, which results in forces that cancel out, at least partially, those in the excitation. A TMD is most effective when the source of the excitation is narrow-band (i.e., at a single frequency). A simple design for a TMD that can be integrated into an existing composite floor system with minimal effort or interference with other building systems. Parameters affecting the TMD performance are also investigated.

## TUNED-MASS DAMPER THEORY

The basic dynamic system that illustrates the theory of the TMD is the two-degree-of-freedom (2-DOF) lumped-mass model shown in Figure 1. The floor system is modeled as a mass, $m_{\text{Floor}}$, and stiffness, $k_{\text{Floor}}$, and the TMD is a smaller mass, $m_{\text{TMD}}$, attached to the floor with a spring of stiffness $k_{\text{TMD}}$. A harmonic force acts on the floor system (but not on

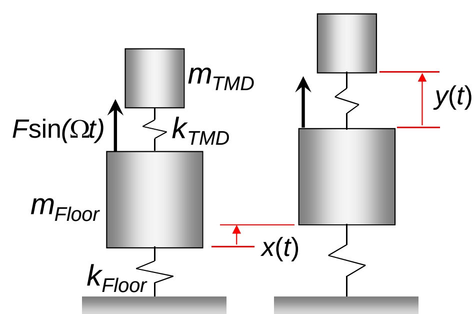


**Figure 1 Simple Model for a Tuned-Mass**

the TMD) with force amplitude, $F$, and frequency $\Omega$. The absolute displacement of the floor system is $x(t)$ and the displacement of the TMD mass relative to the floor is $y(t)$. The objective is to determine properties for the TMD ($m_{\text{TMD}}$ and $k_{\text{TMD}}$) that minimize the response of the floor; i.e., minimize $x(t)$, with the given forcing function.

The equations of motion are two coupled second-order ordinary differential equations (ODEs) given by

$$\begin{aligned}(m_{Floor}+m_{TMD})\ddot{x}+m_{TMD}\ddot{y}+k_{Floor}\,x&=F\sin(\Omega t)\\ m_{TMD}\ddot{x}+m_{TMD}\ddot{y}+k_{TMD}\,y&=0\end{aligned} \qquad (1)$$

and the steady-state solution of these equations yields the expressions for the floor and TMD displacements:

$$x(t)=\frac{(k_{TMD}-m_{TMD}\Omega^2)F\sin(\Omega t)}{m_{Floor}m_{TMD}\left[\Omega^4-\left(\frac{k_{TMD}}{m_{TMD}}+\frac{k_{TMD}}{m_{Floor}}+\frac{k_{Floor}}{m_{Floor}}\right)\Omega^2+\frac{k_{Floor}k_{TMD}}{m_{Floor}m_{TMD}}\right]} \qquad (2)$$

$$y(t)=\frac{m_{TMD}\Omega^2F\sin(\Omega t)}{m_{Floor}m_{TMD}\left[\Omega^4-\left(\frac{k_{TMD}}{m_{TMD}}+\frac{k_{TMD}}{m_{Floor}}+\frac{k_{Floor}}{m_{Floor}}\right)\Omega^2+\frac{k_{Floor}k_{TMD}}{m_{Floor}m_{TMD}}\right]}$$

The denominators for the two responses are identical. The term in the [ ] goes to zero for two distinct values of $\Omega = \Omega_1$ and $\Omega = \Omega_2$. These are the two resonance frequencies for this 2-DOF system, and when the excitation frequency is equal to either of these, the response amplitudes of the floor and TMD grow without bound. If damping had been included, the response amplitudes would grow to some finite limit.

The important feature of these equations is found in the two numerators. The numerator of the floor system response contains the term ($k_{TMD}$ - $m_{TMD}\Omega^2$), which is zero when

$$\sqrt{\frac{k_{TMD}}{m_{TMD}}}=\Omega \qquad (3)$$

Apparently the floor system motion at frequency $\Omega$ can be completely eliminated by attaching any size mass to the floor, as long as the stiffness of the connecting element is determined from Equation (3). The motion of the TMD is not zero, however, which leads to the familiar description that the TMD "absorbs" the vibration: the TMD vibrates, but the floor remains stationary.

The non-intuitive finding that a negligible TMD mass can absorb 100% of the motion of any floor system is a consequence of not including damping in these basic equations. The presence of damping limits the effectiveness of the TMD. Some level of vibration at $\Omega$ will always remain, but it can be significantly reduced compared to the floor system's response without the TMD. The mass of the TMD is also a critical player as discussed in the following sections.

Equations (2) and (3) show that a single TMD is only effective at mitigating the response when the excitation is harmonic and the TMD is tuned [Equation (3)] to that frequency. Single-frequency sources of excitation are rare, but some common forms of excitation possess narrow band tones; e.g., vortex shedding, walking/running, and rotating-element machinery. Equation (2) indicates that a TMD is potentially effective as long as the TMD is tuned to the excitation frequency, but the excitation frequency is arbitrary—it need not coincide with a floor system resonance frequency, although that condition will cause the greatest floor vibration.

## COMPOSITE FLOOR VIBRATION CHARACTERISTICS

Four representative composite floor systems are designed and presented in Reference (a). One of those systems, Case 2, is adopted here as the baseline floor system for this study [Figure 2]. The metal deck is 3-in deep and the total depth of the normal-weight concrete ($f_{c'}$ = 3 ksi) is 3 in.

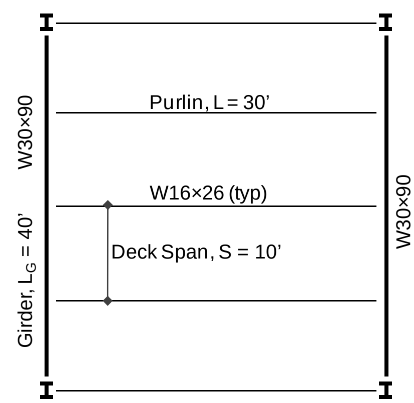


**Figure 2 Typical Composite Floor System**

The first four resonance frequencies and mode shapes of the purlin that frames into the midspan of the girder are shown in Figure 3 [from Reference (a)]. The simple mass-spring model of a floor system discussed above only has one resonance frequency, but the much more realistic model used to obtain these results provides a theoretically infinite number of resonance frequencies with unique mode shapes.

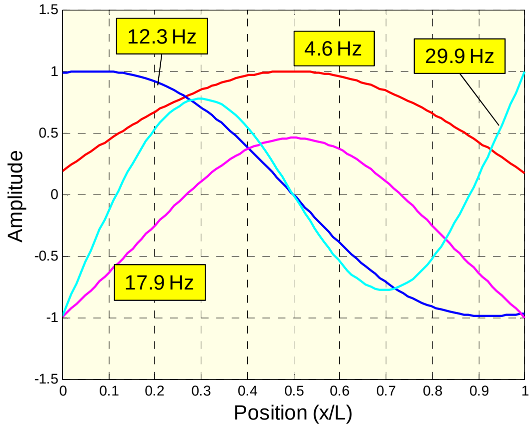


**Figure 3 Purlin Resonance Frequencies and Mode Shapes**

A harmonic source of excitation can directly excite any one of the resonant modes of a structure and cause disturbing, if not damaging, levels of vibration. However, it is progressively more difficult to elicit a perceptible response from the higher modes because they require more strain energy and people tend to be most sensitive to vibration in the 4- to 8-Hz range [Reference (b)]. Most structural vibration problems are related to the lowest few modes. In this case, the 4.6-Hz mode (the fundamental mode) is of primary interest because it is easiest to excite and that resonance frequency falls in the high motion-sensitivity band.

### Harmonic Response of a Representative Floor System

The response of a structure to harmonic excitation is most conveniently represented in the form of a frequency response function (FRF), H(Ω). A harmonic force is applied at one location ($x = x_F$) and the steady-state response of interest (e.g., displacement, acceleration, stress, etc.) is measured at another location ($x = x_R$). The FRF is the ratio of the response magnitude to the input (force) magnitude at the excitation frequency Ω. This scenario is illustrated in Figure 4, where the force acts at $x_F$ and the acceleration is obtained at $x_R$.

Two FRFs are plotted in Figure 4. The red curve is obtained with the force acting at midspan and the acceleration measured at midspan ($x_F = x_R = 0.5L$), and the blue curve is obtained with the force applied at $x_F = 0.25L$ and the acceleration measured at $x_R = 0.75L$. The FRF magnitude is given in terms of the number of g's of acceleration per pound of applied force. The FRF scales linearly, hence, a 10-lbf force results in ten times the acceleration.

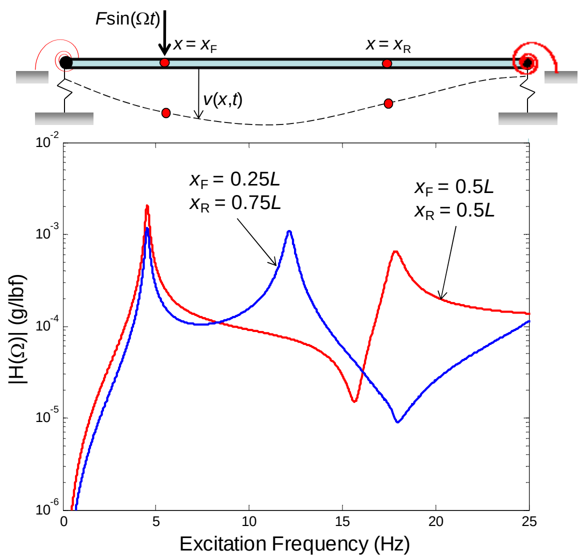


**Figure 4 Frequency Response Function for Composite Beam (W16×26)**

The peaks in the FRF curves correspond to the resonance frequencies at 4.6, 12.3, and 17.9 Hz; however, there are only two peaks in each FRF. There is no peak in the red curve at 12.3 Hz (second mode) because the acceleration is obtained at $x_R/L = 0.5$ and the second mode shape has a node (zero magnitude) at $x/L = 0.5$ [Figure 3]. Likewise, there is no peak in the blue curve at 17.9 Hz (third mode) because the third mode shape is essentially zero at $x/L = 0.75$. Note that the peaks are bounded (response is not infinite) at the resonance frequencies because 2% damping is included for all of the modes in this model.

The largest response is obtained when the harmonic force acts at midspan and with a frequency that coincides with the first mode resonance (Ω = 4.6 Hz) and the acceleration is also measured at midspan. In this case, a 1-lbf force magnitude generates an acceleration magnitude of 0.0021g. A response level of 0.005g is perceptible, while 0.5g is clearly disturbing. Force magnitudes in 10- to 100-lbf range can cause problematic levels of vibration (i.e., 0.02 to 0.2g).

#### Force-Time History: Walking

A person walking across a floor applies forces to the floor with each step. Walking is not a harmonic source of excitation, but it is periodic because each step occurs at a regular interval of about $\Delta t \approx 1$ sec. Periodic excitation may be thought of as multiple instances of superimposed harmonic excitation at frequencies of $\Omega_0 = 1/\Delta t$ Hz and its harmonics at $\Omega_1 = 2/\Delta t$ Hz, $\Omega_2 = 3/\Delta t$ Hz, etc. Depending upon the actual step interval, the third, fourth, or fifth harmonic can coincide with the fundamental resonance frequency of a composite floor system.

Approximations of the force-time history for a step are presented in References (a) and (b) and are reproduced in Figure 5 as five successive steps. The triangular step is adapted from Reference (b) and the smooth-curve step is defined in Reference (a) with sine (increasing force) and cosine (decreasing force) functions. Both steps have identical amplitudes of 400 lbf, step intervals, $\Delta t = 0.9$ sec, and contact durations (the time each foot remains in contact with the floor) of $t_d = 1.3$ sec. The faster initial rise in force of the triangular curve might result from a hard-sole shoe on a hard surface like a tile floor, whereas the smooth curve might result from the same person wearing soft-sole shoes on a carpeted surface.

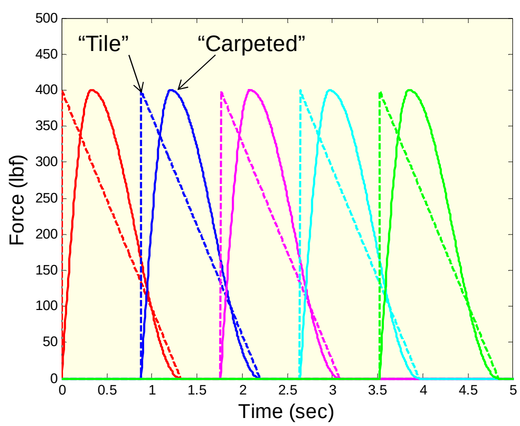


**Figure 5 Representative Walking Force-Time Histories**

The force amplitude associated with each harmonic is determined from the Fourier Series coefficients and these are plotted in Figure 6. Most of the force is concentrated at the fundamental frequency ($f_0 = 1.14$ Hz) and decreases with each successive harmonic. The force amplitude of the fourth harmonic ($f_3 = 4.5$ Hz) is of most interest here because that harmonic coincides with the first mode resonance frequency of the floor system and therefore has the greatest potential for exciting the floor system's resonant mode and generating a potentially undesirable level of vibration.

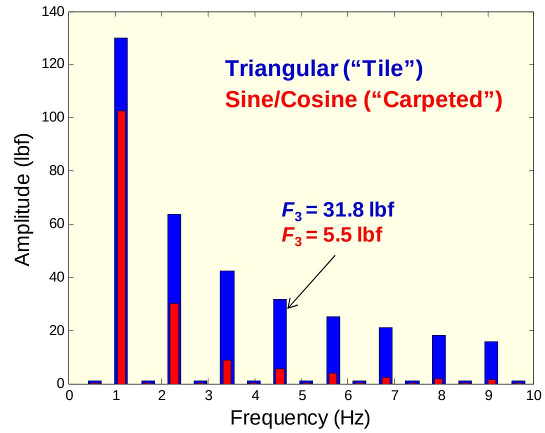


**Figure 6 Amplitudes of Harmonics in the "Walking" Force-Time Histories**

The rise rate of the force has a significant effect on the magnitude of each harmonic. An abrupt application of force, such as the "tile" force-time history, produces higher-amplitude harmonics than does a more gradual application of the force (e.g., the "carpeted" force-time history).

The FRFs for this floor system suggest that force amplitudes in the 10- to 100-lbf range at the first mode resonance frequency can produce potentially annoying levels of vibration. The "tile" step produces 31.8 lbf at the fourth harmonic, whereas the "carpeted" step results in only 5.5 lbf at the fourth harmonic. Hence, one might expect higher-than-desirable vibration for the "tile" step.

#### Forced Response of Floor System

The multi-degree-of-freedom (MDOF) dynamics model developed in Reference (a) is used to predict the acceleration at midspan caused by someone walking nearby. The MDOF model includes six modes (all modes with resonance frequencies up to 100 Hz). Modal damping of 2% is included for all of the modes. Real-world damping levels are likely to be somewhat higher and tend to increase with resonance frequency, but the relatively small level of damping included here allows the vibration to persist longer than it might in reality. The excitation is defined as the superposition of ten successive steps, using either the "carpeted" or the "tile" step function. A fourth-order Runge-Kutta numerical integration algorithm is used to solve the six coupled ordinary differential equations.

The acceleration response to the "carpeted" step function is provided in Figure 7. Two horizontal dashed lines are drawn at ±0.005g, which roughly correspond to the acceleration amplitude people can perceive and, possibly, be disturbed by depending upon the duration of the motion and the nature of the individual's work. The fourth harmonic in the excitation drives the fundamental mode of the floor system giving rise to a response amplitude slightly larger than the threshold in this case.

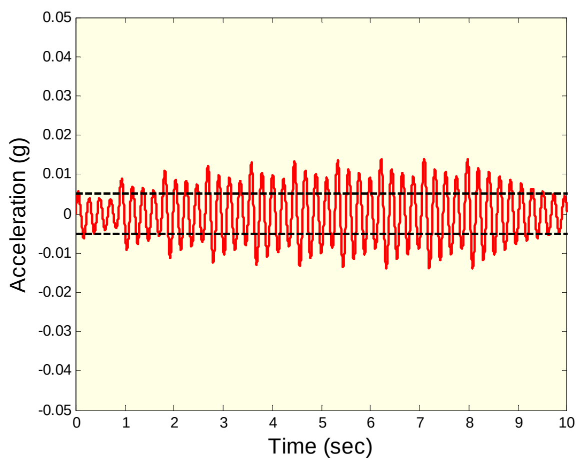


**Figure 7 Acceleration Response at *x* = 0.5*L* Due to Walking on a "Carpeted" Floor**

The analysis is repeated using the "tile" step function, but all other parameters are identical. The acceleration response is shown in Figure 8 and is about ten times larger than that in Figure 7. This comparison is expected, at least to some degree, because the fourth harmonic for the "tile" floor is about six times larger than that of the "carpeted" floor. The other harmonics also contribute to the response which accounts for the difference.

The response of the "tile" floor is large enough that others on the same floor could be annoyed by the vibration. The problem is further exacerbated in a high-traffic area where several people may be walking together creating a larger combined excitation. A tuned-mass damper is investigated to determine if these response levels can be reduced.

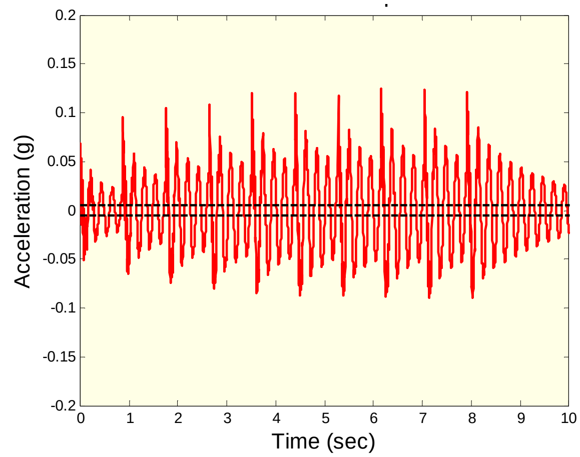


**Figure 8 Acceleration Response at *x* = 0.5*L* Due to Walking on a "Tile" Floor**

## IMPLEMENTATION OF A TUNED-MASS DAMPER

A tuned-mass damper is expected to provide significant mitigation of the vibration environment in this application because the floor system is driven by one of the harmonics in the excitation. The simple 2-DOF model of the TMD discussed above cannot properly describe the more complex motion the elastic beam is capable of. The "floor system" in the 2-DOF model can only displace up and down. The displacement of the beam is the superpostion of its various modes of vibration and varies all along its length. Also, the excitation of interest here has multi-frequency content rather than a single frequency.

A beam model with two TMDs is illustrated in Figure 9. More than one TMD can be attached to the beam. In addition to the mass and stiffness of the TMD, its location must also be considered. Each TMD adds a degree of freedom to the MDOF model. In this case, the displacement of the TMD mass relative to the deflected shape of the beam is used as the TMD degree of freedom. The TMDs in Figure 9 are idealized as lumped (point) masses, which may be reasonable depending upon how the TMD is actually constructed.

The effect of a single TMD mass (400 lbm) placed at midspan of the composite beam is shown in the FRF plotted in Figure 10. Also plotted in the figure is the FRF for the beam with no TMD [from Figure 4]. In this case, the TMD is tuned to 4.5 Hz, which is very close to the first mode resonance frequency. The ideal placement for the TMD is where the amplitude of the first mode shape is a maximum (positive or negative), which is at midspan in this case.

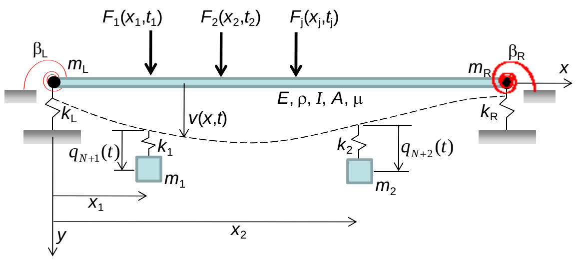

**Figure 9 Floor System Beam Model With Lumped Mass TMDs**

### Tuned-Mass Damper Design

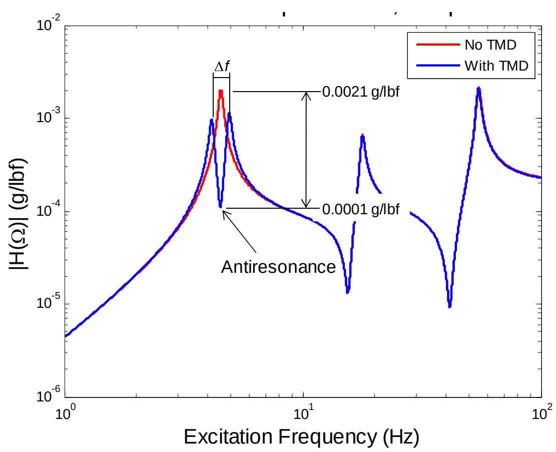


**Figure 10 Effect of TMD on FRF**

The TMD does not "eliminate" the resonant mode at 4.6 Hz; instead, it creates two modes with an antiresonance between them at the tuned frequency of 4.5 Hz. The antiresonance is created by the two closely-spaced modes being nearly 180 deg out of phase with each other. If there were no damping, the two modes would be 180 deg out of phase and the FRF value at the antiresonance would be zero.

The mass of the TMD controls the frequency difference between the two modes, $\Delta f$, which also affects the response ratio—the ratio of the FRF magnitude at 4.5 Hz before adding the TMD to the FRF magnitude at the antiresonance. If the excitation were in fact harmonic (i.e., a single frequency at 4.5 Hz), the response would be .0021/.0001 = 21 times less with the TMD present. The frequency difference, $\Delta f$, is important because real-world excitation frequencies can vary and a larger $\Delta f$ reduces the system's sensitivity to this variation. The frequency difference and response ratio for the composite beam are plotted in Figure 11 for practical TMD weights ranging from 50 lbf to 1000 lbf.

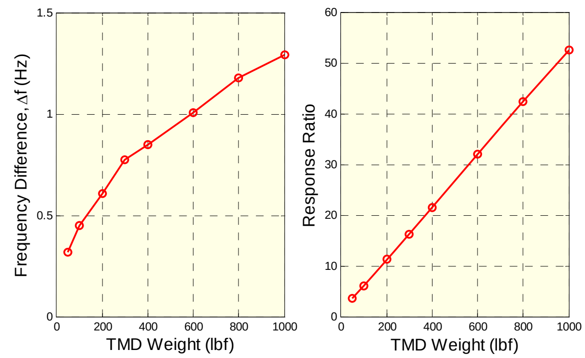


**Figure 11 TMD Performance VS TMD Weight**

The scenario of interest in this paper requires that a TMD be added to an already-existing floor system. Hence, the amount of added weight that is permissible is limited, unless accompanied by a potentially extensive reinforcement effort. Another practical consideration is the distance that the TMD can extend below the bottom of the existing beam because of the ceiling for the floor below. These issues, and others, affect the final design and performance of the TMD.

In the interest of minimizing the distance that the TMD projects below the beam, a flexure-based design is selected for this application. In effect, two equal-weight TMDs are placed at midspan as shown in Figure 12. A 45"×6"×1/2" plate is bolted to the bottom flange of the beam and serves as the flexure (cantilever beam) for the two identical TMD masses. The TMD masses are assembled from steel plates joined to a HSS7×2×1/4 section welded at the free end of the flexure.

Each TMD weight is 200 lbf and the dimensions of the flexure plate are selected to provide a fundamental cantilever resonance frequency of 4.5 Hz. The offset location of the TMD mass center of gravity, the mass moment of inertia, and the TMD mass are considered when determining the resonance frequency (i.e., the mass is not assumed to be a point mass).

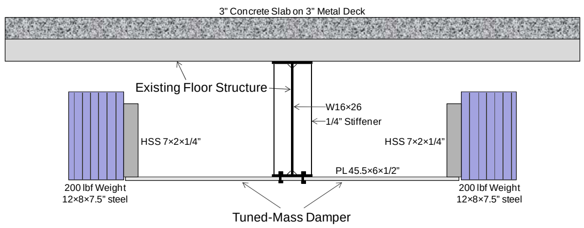


**Figure 12 Flexure-Based Dual TMD**

## Floor System Response with the Tuned-Mass Damper

The MDOF dynamics model is extended to include the two cantilever TMDs and the forced response analysis is repeated to ascertain their effect. The response acceleration for the "carpeted" floor is plotted in Figure 13. Responses for a single 400-lbf point mass (blue) (*a la* Figure 9) and for two 200-lbf cantilever TMD masses (red) (*a la* Figure 12) are provided. The amplitude of the peak acceleration is about 0.004g, which is reduced from 0.014g with no TMD (Figure 7). Here, the response ratio is 3.5—the TMD reduces the response by a factor of 3.5 compared to the original beam with no TMD.

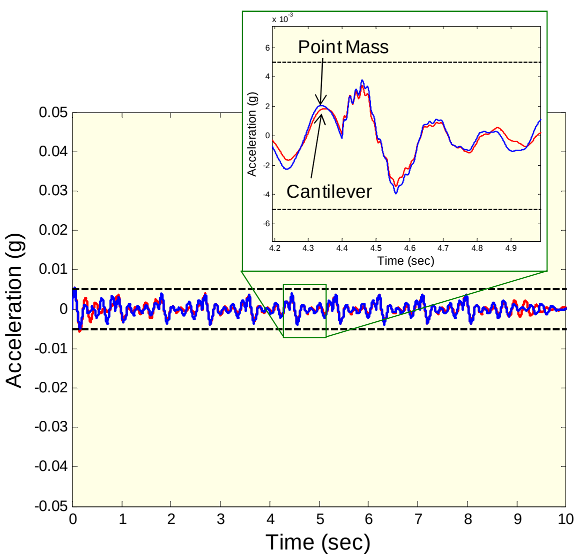


**Figure 13 Acceleration Response With TMD at *x* = 0.5*L* Due to Walking on a "Carpeted" Floor**

Several points are worth noting. First, while it is essential to use a sufficiently complex model to properly predict the resonance frequency of the cantilever TMD, it is not necessary to include that more complex model in the complete system analysis model—a spring and point mass model is adequate for composite floor system analysis.

The response ratio is misleading in this case because the excitation is not a pure harmonic; it contains many harmonics. A better method for assessing TMD performance is with the power spectrum shown in Figure 14, where the effect in the frequency range of interest can be seen more readily. A 24.7-dB reduction in the vibration at 4.5 Hz is obtain, which translates into a response ratio of 17 (close to the 21 predicted from the FRFs [Figure 10] for harmonic excitation).

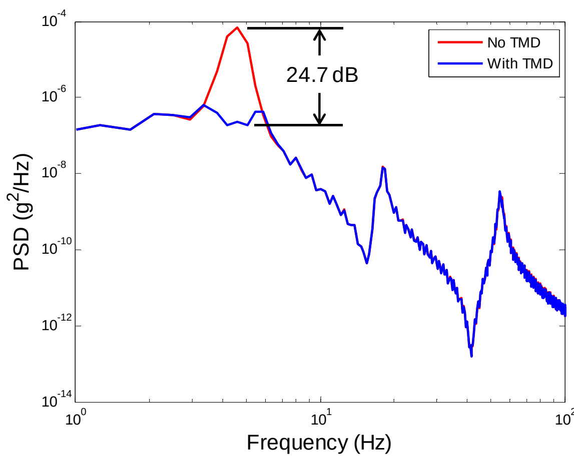


**Figure 14 Acceleration Power Spectra Comparison for "Carpeted" Floor**

The effect of the harmonics is even more pronounced for the "tile" floor. The response of the beam with the two cantilever TMDs is shown in Figure 15. The peak response is 0.062g, which is only down by a factor of 2, but the power spectrum comparison shown in Figure 16 shows the same 24.7 dB reduction in the vibration achieved with the response of the "carpeted" floor. The response level, however, is still 5.8 times larger at 4.6 Hz for the "tile" response because of the relative magnitudes of the fourth harmonic [Figure 6].

The zoomed-in portion of the response in Figure 15 shows that a significant contributor to the motion occurs with a period of 1.9 ms, which is the 54-Hz resonant mode of the beam [Figure 10]. People are most sensitive to vibration in the 4- to 8-Hz range, so this motion may not be felt. The sharp peaks occur at each step (at 1.1 Hz). Also, the extreme nature of the "tile" step force rise time exaggerates the magnitude of the higher harmonics. Finally, it worth noting that this high frequency response is a consequence of the more detailed beam model used here instead of the grossly simplified model shown in Figure 1.

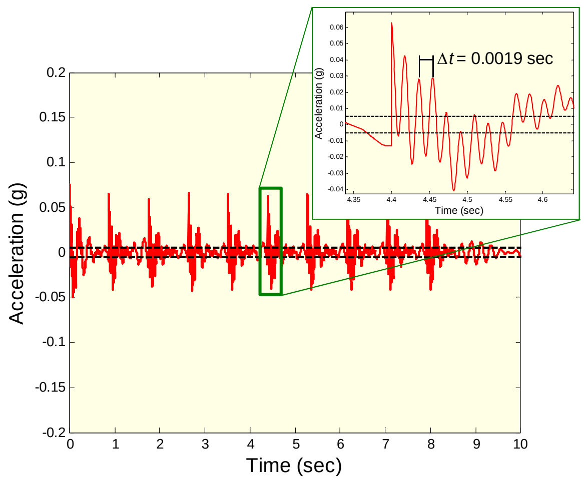


**Figure 15 Acceleration Response With TMD at *x* = 0.5*L* Due to Walking on a "Tile" Floor**

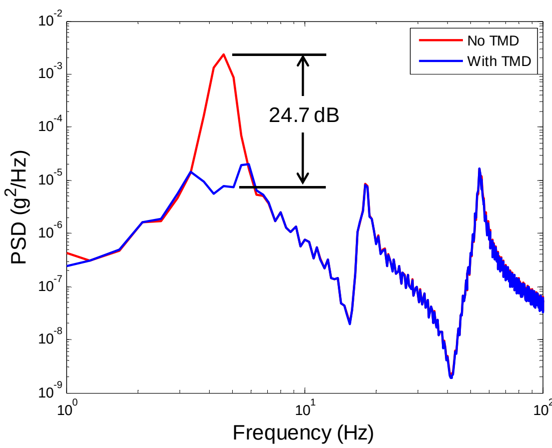


**Figure 16 Acceleration Power Spectra Comparison for "Tile" Floor**

The implementation of the TMD for the "tile" floor is likely successful from a perceived motion perspective. However, for the sake of discussion, one may wonder if adding another TMD tuned to 54 Hz would be effective in mitigating the high frequency oscillation. Two additional TMDs are placed at x/L = 0.2 and 0.8 (where the 54-Hz mode shape has the greatest magnitude). A TMD weight of 200 lbf each (400 lbf total) is used and both are tuned to 54 Hz. The resulting FRF and zoomed-in acceleration response are shown in Figure 17.

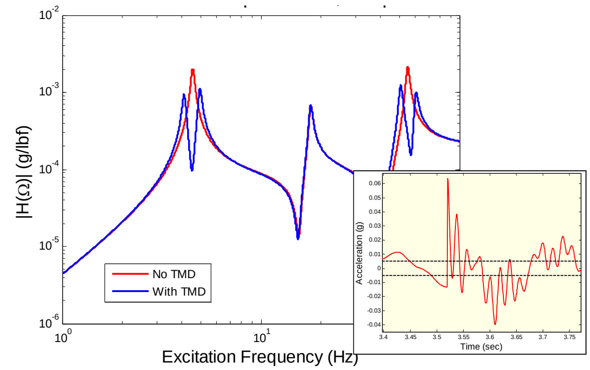


**Figure 17 FRF and Response for TMDs at x/L = 0.2, 0.5, and 0.8**

### Structural Performance of the Cantilever TMD

An additional antiresonance is now present at 54 Hz as desired. The peak response does not change; however, but the number of 54-Hz oscillations is reduced. While this example shows that it is possible to add additional TMDs to the beam, the perceptible difference appears to be negligible in this case.

The dynamic design and performance of the TMD on the composite beam is the focus of the preceding discussion; however, a complete design requires that the cantilever structure also be assessed for its own structural integrity—it must be capable of surviving the static and dynamic loads produced during service. A schematic of the TMD is shown in Figure 18.

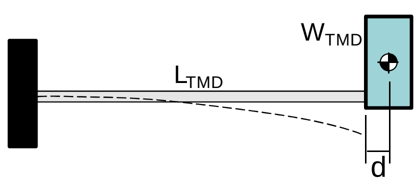


**Figure 18 Cantilever TMD**

The static stress and tip deflection are given by

$$\sigma_{Static} = \frac{6W_{TMD}\left(L_{TMD}+d\right)}{bt^2} \qquad \delta_{Static} = \frac{4W_{TMD}\left(L_{TMD}+d\right)^3}{Ebt^3} \tag{4}$$

where $W_{TMD}$ = 200 lbf, $L_{TMD}$ = 20.5 in., $d$ = 3.8 in, $b$ = 8 in., and $t$ = 0.5 in., which yields a static stress of 14.6 ksi and a static deflection of 0.4 in.

The dynamic contributions are given by

$$\begin{aligned} \sigma_{Dyn} &= \frac{1}{2} Et\left|q_{TMD}(t)\right|\psi''_{TMD}(y_{TMD}) \\ \delta_{Dyn} &= \left|q_{TMD}(t)\right|\psi_{TMD}(y_{TMD}) \\ \psi_{TMD}(y) &= a\sin(\kappa y) + b\cos(\kappa y) + c\sinh(\kappa y) + d\cosh(\kappa y) \end{aligned} \tag{5}$$

where $a = -c$ = 3.546, $-b = d$ = 3.112, $\kappa$ = 0.0317 1/in., $y_{TMD}$ = 0 in. for stress and 20.5 in. for displacement, and $|q_{TMD}|$ is the magnitude of the TMD degree of freedom displacement = 0.0077 in., which is also the dynamic displacement because $\psi_{TMD}$ is normalized to unity.

The dynamic stress is plotted in Figure 19 and is quite low compared to the static stress. The total displacement of the TMD is about 0.5 in. and the total stress in the A36 plate (the flexure) is about 15 ksi. Both the stress and the deflection for the cantilever TMD are within reasonable limits. The total force transmitted to the existing steel beam is the TMD weight plus the dynamic load of 8.4 lbf (from $V = 2EI\left|q_{TMD}\right|\psi'''_{TMD}(0)$) is less than 425 lbf, which is low enough to avoid needing to strengthen the beam.

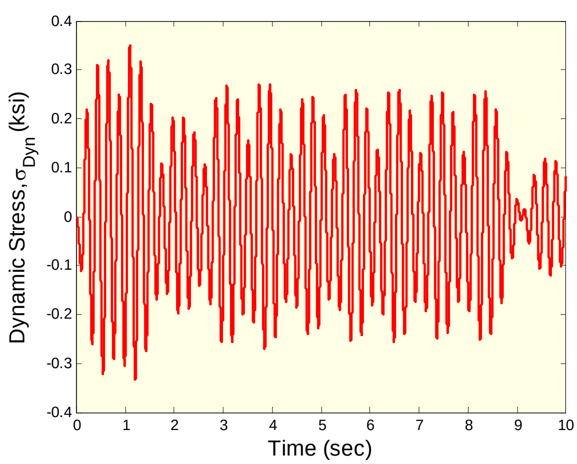


**Figure 19 Dynamic Stress in the TMD**

## CONCLUSIONS

A low-profile flexure-based TMD is designed and shown to mitigate the walking-induced vibration of a preexisting composite beam. The TMD is ideally used to control the vibration caused by a pure harmonic source of excitation. Walking is periodic excitation comprised of many superimposed harmonics. The third, fourth, or fifth harmonics (3 to 5 Hz) fall in the frequency range that coincides with typical composite floor system resonance frequencies and the frequency band in which people are most sensitive to vibration.

A practical flexure-based TMD (a cantilevered plate with a tip mass) is designed to mitigate the vibration predicted using a MDOF dynamics model of a representative composite beam. Design constraints include the desired vibration mitigation, a limit on the added weight of the TMD, the space required below the existing steel beam, and the TMD's own structural adequacy. If the TMD weight is too large, the supporting steel beam would have to be strengthened. On the other hand, TMD effectiveness increases with its mass. The TMD cannot protrude too far below the existing beam to avoid interfering with the suspended ceiling for the floor below. This TMD only requires 1 in. of clearance below the existing beam and is easily installed via a simple bolted connection to the bottom flange of the steel beam. Finally, the combined static and dynamic forces in the TMD are shown to be well within its structural capability.

Two cantilever TMDs, each weighing 200 lbf, are attached to the bottom flange of the steel beam and numerical simulations of the beam's response are performed using a dynamics model developed for this study. Power spectral analysis shows that the vibration amplitude near the sensitive 4.5 Hz region is reduced by 24.7 dB (a factor of 17) with the TMDs, regardless of the floor hardness (carpet VS tile). While the TMD effectiveness is the same, the vibration response for the "tile" floor is about 6 times higher than for the "carpeted" floor because of the larger harmonic amplitudes.

## REFERENCES


(a) Lamb, J. L., "Walking-Induced Vibration of Composite Floor Systems," SEI Technical Report, March 2011.

(b) Naeim, F., "Design Practice to Prevent Floor Vibrations," Structural Steel Education Council, September 1991.