Using a Pendulum Tuned-Mass Damper to Control Walking-Induced Motions of an Interior Flexible Suspended Walkway

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ABSTRACT

Flexible walkways and pedestrian bridges are susceptible to lateral walking-induced excitation. Walking produces vertical excitation at a fundamental excitation frequency near 1 Hz, but the fundamental lateral excitation frequency is near 0.5 Hz. The first lateral bending mode of the walkway considered in this paper has a resonance frequency of 0.56 Hz. Two forms of tuned-mass dampers are shown to be very effective at reducing the lateral walking-induced excitation of the 60-ft span mezzanine walkway. The pendulum is not quite as effective as the more traditional mass/spring TMD, but the exposed 3- to 4-ft-long pendulum can contribute to the visual impact of the walkway. The mass/spring TMD reduces the critical vibration by a factor of 50 and can be hidden from view, which has advantages in some applications.

INTRODUCTION

Interior suspended walkways can be practical and provide a dramatic visual. Thin walkways further enhance the dramatic impact, but the reduced stiffness can pose problems because of their higher susceptibility to walking-induced motion. Engineers can mitigate these concerns to some degree via effective design, but there may cases where the proposed structural scheme is at odds with the architectural objectives. More sophisticated engineering solutions may be necessary to provide the architectural impact and a user-friendly structure.

Tuned-mass dampers (TMDs) have been used to control the undesirable vibration in many structures. They are sometimes incorporated as part of the initial design and, in other cases, are added to existing structures to address an unexpected vibration problem that arises in service. TMDs might be thought of as an "enabling technology" because they allow more flexible structures to be considered during the concept design phase, which allows Architects to realize their vision with fewer restrictions.

A 60-ft span flexible walkway is considered in this paper and external and/or internal TMDs are investigated to control the relatively large lateral walking-induced vibrations. The fundamental lateral mode of the walkway occurs at 0.56 Hz, which coincides with the lateral forces generated during walking. An external (i.e., visible) pendulum TMD is designed as one option. An alternative design, a point-mass TMD concealed within the envelope of the walkway structure, is also considered. Numerical simulations with both TMD options show both designs are effective at providing significant mitigation of walking-induced lateral motions. The external pendulum, which swings back and forth below the walkway, may be more appropriate for educational venues or work environments that emphasize artistic expressions of creativity (e.g., advertising and architectural firms). The internal TMD may be more appropriate for work environments that emphasize deliberative decision-making (e.g., financial and legal firms). The geometric flexibility and effectiveness of the TMD allow for a wider variety of structural forms that might otherwise be rejected due to excessive flexibility.

WALKWAY DESIGN

The walkway of interest here is illustrated in Figure 1. The walkway is simply supported at each end, but is supported vertically at midspan by two ½"-diameter 7-wire steel strands. The wire supports provide vertical support but offer very little stiffness laterally. The strand's connection to the walkway is provided by a W14×43 oriented orthogonal to the walkway. The primary structure of the walkway is also comprised of two W14×43, which are fully welded to W14×43 cross member at midspan.

Cedar planks are used for the walking surface and ¼"-thick plywood provides an all-around wood veneer to conceal the structural steel beams.

Neither the planks nor the plywood participate in the primary load-carrying capacity of the walkway. The planks are connected to the top flanges of the W14×43 beams, but do not provide lateral bracing of the beams because the individual planks do not act as a diaphragm.

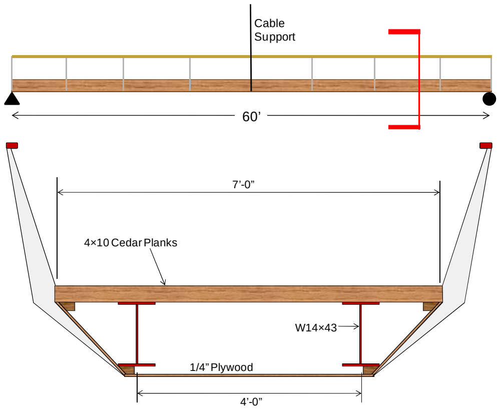

Figure 1 Walkway Span and Cross-Section

The walkway is designed for a 100-psf live load, self weight, and the superimposed dead load, which includes some miscellaneous mechanical equipment. The bending moment diagrams resulting from live load acting on one side of the cables and on the entire walkway surface are plotted in Figure 2. The supporting steel beams are designed for a factored moment of 60 k-ft with an unbraced length of 25 ft for partial live loading and 84 k-ft and an unbraced length of 15 ft for full live load.

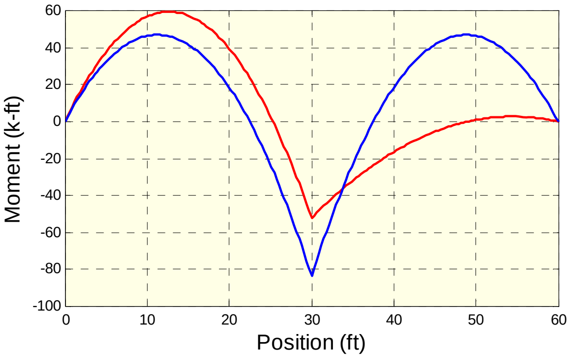

Figure 2 Bending Moment Distributions

The structural members must also satisfy the static deflection criteria. The deflected shape is shown in Figure 2 for the case where the service live load acts on half of the 60-ft span. The W14×43 sections shown in Figure 1 are more than adequate to satisfy the strength and static serviceability requirements and are also compatible with the maximum overall walkway depth allowed for aesthetic considerations.

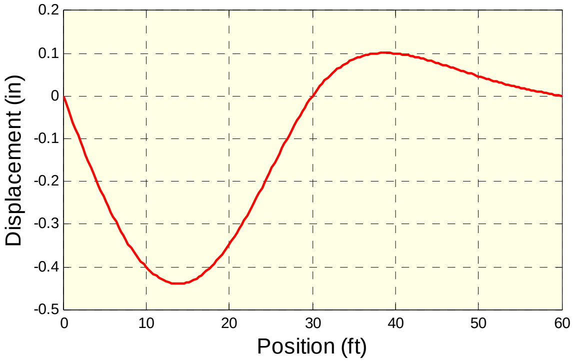

Figure 3 Vertical Deflection with Partial Loading

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WALKWAY DYNAMICS ASSESSMENT

The walkway satisfies the strength and serviceability requirements in the Building Code, but additional analysis is required when the dynamic characteristics of a structure are suspect, as they are in this case. The simple structural form of the walkway permits development of a relatively simple dynamics model. In general, the walkway can deflect laterally and vertically, and rotate about its axis. The elastic deflection is described by $v(x,t)$, $w(x,t)$, and $\phi(x,t)$ for the lateral, vertical, and rotational deflections, respectively as shown in Figure 4.

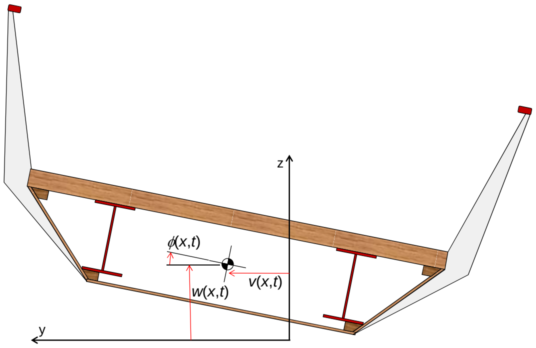

Figure 4 Deflection of a Given Cross-Section

The unknown functions of axial position along the walkway, x, and time, t, are further discretized for a $3N$ degree-of-freedom model using sinusoidal assumed mode shapes as shown in Equations (1). The unknowns are the $3N$ time-dependent functions $q_n(t)$. The number of assumed modes used for each deflected shape is determined by the maximum resonance frequency desired. In this case, $N = 15$ provides all of the modes with resonance frequencies up to 100 Hz.

$$v(x,t) = \sum_{n=1}^{N} q_n(t)\,\sin\!\left(n\pi x/L\right) \tag{1}$$
$$w(x,t) = \sum_{n=1}^{N} q_{n+N}(t)\,\sin\!\left(n\pi x/L\right)$$
$$\phi(x,t) = \sum_{n=1}^{N} q_{n+2N}(t)\,\sin\!\left(n\pi x/L\right)$$

The two support cables at midspan act as springs that resist lateral and vertical translation and rotation. The stiffness provided by each cable is given by

$$k_v = \frac{T}{L_C}, \quad k_w = \frac{EA_C}{L_C}, \quad k_\phi = \frac{EA_C d}{L_C} \tag{2}$$

where T is the service load tension in the cable (5 to 18 kips, depending upon the proportion of live load on the walkway), L_C is the length of the cable (30 ft), A_C is the cross-sectional area (0.153 in^2), and d is the horizontal distance from the center of the walkway to the cable attachment point (5 ft).

The deformation kinematics for each beam are defined using Equations (1) and incorporated into the definitions of the system kinetic and potential energy. Warping deformation is ignored. Lagrange's Equations are used to derive the equations of motion and are linearized assuming small displacements and rotations. The assumed mode shapes are orthogonal, so the linearized equations only have off-diagonal terms in the stiffness matrix resulting from the springs at midspan. The eigenvalues (resonance frequencies) and eigenvectors (mode shapes) are obtained from the diagonal mass matrix and the symmetric stiffness matrix.

The first five modes are summarized in The lateral sidesway mode has the lowest resonance frequency at 0.56 Hz. The first mode can vary from 0.43 Hz to 0.70 Hz as the live load varies from 100% to 0%, respectively. The resonance frequency increases as the percentage of the live load decreases because the live load represents superimposed mass. Live load also increases the lateral stiffness offered by the support cables, but this effect is small compared to that of the added mass.

Table 1 for the walkway loaded with 25% of the design live load. The lateral sidesway mode has the lowest resonance frequency at 0.56 Hz. The first mode can vary from 0.43 Hz to 0.70 Hz as the live load varies from 100% to 0%, respectively. The resonance frequency increases as the percentage of the live load decreases because the live load represents superimposed mass. Live load also increases the lateral stiffness offered by the support cables, but this effect is small compared to that of the added mass.

Table 1 Summary of First 5 Modes

Mode	f_n	Description
1	0.56 Hz	1st Lateral Bending
2	2.02 Hz	2nd Lateral Bending
3	4.33 Hz	1st Vertical Bending
4	4.55 Hz	3rd Lateral Bending
5	6.22 Hz	2nd Vertical Bending

The 0.56-Hz lateral mode is particularly problematic because walking-induced forces can directly excite this mode. The largest forces generated while walking act in the vertical direction and therefore tend to excite the vertical motion of the walkway. Walking also produces lateral forces. Each step pushes outward on the floor; the left foot pushes to the left and the right foot pushes to the right. If each successive footfall occurs at a 1-sec interval, the fundamental lateral excitation frequency is 0.5 Hz. If each footfall occurs at a 0.9-sec interval, the lateral excitation frequency coincides with the first mode of the walkway.

Interestingly, the fundamental vertical excitation frequency is 1.1 Hz (each footfall pushes down on the floor, so the frequency is twice that of the lateral excitation), and the fourth harmonic of the vertical excitation falls at 4.4 Hz, which is very close to the first vertical mode of the walkway. There is a strong possibility that walking-induced forces can directly excite both the lateral and the vertical modes of the walkway.

People are more sensitive to lateral motions as they walk because that motion tends to be destabilizing and can cause a person to lose their balance. An almost identical scenario occurred in 2000 with the original design of the London

Millennium Footbridge that spans the Thames [Reference (a)]. The lowest lateral modes of that bridge fall in the 0.5- to 1.0-Hz range, which are within the lateral pedestrian excitation frequency band as discussed above. At one point, immediately after opening the bridge to traffic, some of the pedestrians became uncomfortable and reflexively grabbed the handrails when lateral accelerations reached magnitudes near 0.2g. The Millennium Footbridge was closed and successfully retrofit with TMDs and viscous dampers, but that experience highlights the critical role that crowd dynamics plays in the excitation of flexible walkways and bridges.

Frequency Response Characteristics of the Walkway

The frequency response characteristics of a structure shows the steady state response at some location on the structure to a given harmonic excitation applied at some location on the structure. The amplitude of the frequency response function (FRF) is presented as the ratio of the response magnitude to the force magnitude. Four FRF curves are plotted in Figure 5 showing the lateral and vertical accelerations a person standing at x_R = 0.4L would feel (as measured 5.5 ft above the walking surface) per pound of force generated by an harmonic force applied in the plane of the walking surface (Fy) or normal to the walking surface (Fz) at x_F = 0.4L.

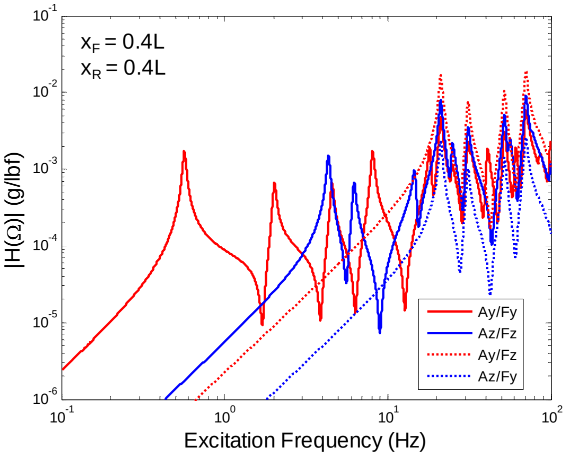

Figure 5 Frequency Response for Excitation at 0.4L and Response at 0.4L

Each peak in a curve corresponds to a resonance frequency of the walkway. The 0.4L positions are selected to avoid placing the excitation and response locations at nodes of the walkway. The Ay/Fy curve is of particular interest in this paper because it conveys the lateral acceleration caused by a lateral force. In this case, where $x_F = x_R$ = 0.4L, the peak at 0.56 Hz yields an FRF magnitude for Ay/Fy = 0.0017 g/lbf. Hence, a total lateral force magnitude of about 120 lbf at 0.56 Hz is sufficient to cause someone to feel 0.2g of acceleration at that frequency.

One person cannot produce 120 lbf of lateral force under normal walking conditions, but a crowd of people walking together on the walkway (i.e., leaving a conference room after a meeting or going to lunch) could easily produce that level of force. The Millennium Footbridge experience shows that people tend to synchronize their steps when walking in crowded conditions.

The lateral and vertical forces produced by walking are not pure harmonic; they are periodic. Periodic excitation has a fundamental frequency of excitation, but the total energy in the periodic excitation is distributed in the fundamental and in the harmonics (integer multiples of the fundamental frequencies). The hardness of the walking surface, among other factors, affects the magnitude of the excitation in the harmonics [Reference (b)]. The hard wood surface of the walkway considered here would likely tend to magnify the walking-induced forces in the fundamental and its harmonics more than, say, a carpeted walking surface would.

Walking-Induced Response of the Walkway

The vertical force-VS-time history for successive steps defined in References (b) and (c) is used in this study as well. The lateral force-time history is also needed for this study. In this case, the lateral force magnitude is taken as 5% of the vertical force. Also, the lateral force associated with the left foot are multiplied by +1, while steps with the right foot are multiplied by -1 to replicate the alternating lateral forces.

The multi-degree-of-freedom (MDOF) dynamics model developed to determine the resonance frequencies and mode shapes of the walkway is expanded to include the walking-induced vertical and lateral forces. The modal superposition technique is used to determine the forced response of the walkway caused by one person walking at midspan. The lateral acceleration 5.5 ft above the walking surface is plotted in Figure 6. In this example, the person is standing near midspan and starts walking at time $t = 0$ sec. The lateral acceleration grows to a peak amplitude of about 0.04g. The increasing acceleration occurs because the first harmonic in the lateral excitation coincides with the first lateral bending mode of the walkway at 0.56 Hz and drives the walkway into resonance. The acceleration magnitude grows to a maximum, which is dictated by the amount of damping in the structure. A uniform 2% critical damping is used for all of the modes in this analysis, which may be high (i.e., will tend to under-predict the peak acceleration levels). Damping levels on the order of 0.6% to 0.8% were measured for the Millennium Footbridge [Reference (a)].

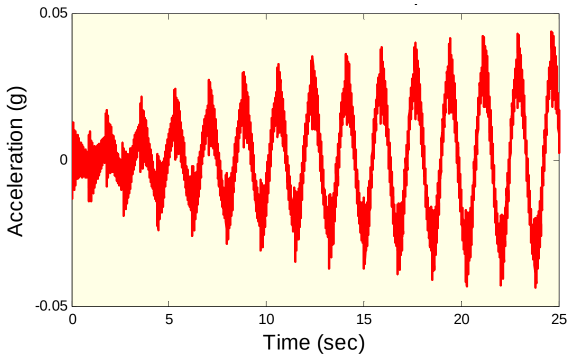

Figure 6 Lateral Acceleration at Midspan

The response shown in Figure 6 suggests that only five people on the walkway are needed to generate the 0.2g lateral acceleration required to cause discomfort. The relatively high level of the live load included in the superimposed mass (25 psf) implies that significantly more than five people are present and that a correspondingly-larger response will result.

The analysis performed here shows that while the structural design depicted in Figure 1 satisfies the strength and static serviceability requirements, the dynamic performance leaves much to be desired. The cause of the undesirable motion is the direct excitation of a resonant mode. A structural redesign with the goal of raising the fundamental lateral mode resonance frequency may be an option, but may result in bulkier structure. The TMD offers an alternative option and is an ideal vibration control solution for this scenario.

TUNED-MASS DAMPER OPTIONS: DESIGN AND PERFORMANCE

The TMD is, in its most basic form, a mass and a spring where the single-degree-of-freedom (SDOF) resonance frequency is adjusted to coincide with the dominant frequency of the troubling motion. In this case, the excessive motion occurs at 0.56 Hz. The TMD mass has a direct influence on the robustness and effectiveness of the TMD [Reference (b)], so maximizing the TMD mass is advantageous. There is, of course, a structural impact associated with the additional mass that must be considered.

Tuned-Mass Damper Kinematics

The additional degree of freedom associated with the TMD must be integrated into the walkway MDOF model to assess its impact. The kinematic assumptions for two different TMDs are illustrated in Figure 7. This figure is meant to be superimposed with Figure 4. Both TMDs are connected to a fixed point (y_T, z_T) on the walkway cross-section located at $x = x_T$. One of the TMDs illustrated in Figure 7 is a traditional spring with stiffness, k_T, and mass, m_T. The spring displacement in the y-direction is $q_T(t)$.

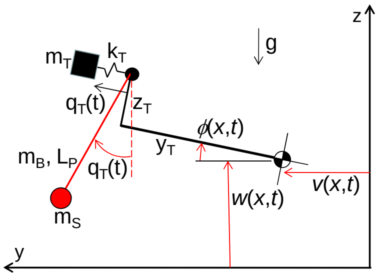

Figure 7 Kinematic Model for Point-Mass and Pendulum TMDs

An alternative form of TMD is shown as a pendulum with its pivot point located at (y_T, z_T). The rotation angle is q_T(t). The restoring force for the pendulum is provided by gravity rather than an elastic spring and the mass is distributed between the bar (length L_P), m_B, and the tip mass, m_S. The SDOF resonance frequencies for the two TMDs shown in Figure 7 are given in Equations (3)

$$f_{TMD} = \begin{cases} \dfrac{1}{2\pi}\sqrt{\dfrac{k_T}{m_T}} \quad \text{Hz} \\ \dfrac{1}{2\pi}\sqrt{\dfrac{\left(\frac{m_B}{2} + m_S\right)L_P g}{\frac{m_B L_P^2}{3} + m_S L_P^2 + I_S}} \quad \text{Hz} \end{cases} \qquad (3)$$

where I_S is the mass moment of inertia of the pendulum tip mass. The mass of the mass/spring TMD is a critical parameter in the TMD resonance frequency, but the pendulum mass plays a minor role for the pendulum. If the pendulum is idealized as a point mass attached to a massless bar, the pendulum resonance frequency reduces to the familiar result: $f_{TMD} = \sqrt{g/L_P}/2\pi$, which is completely independent of the pendulum mass.

More than one TMD can be incorporated into a structure, but only of the forms shown in Figure 7 is added to the walkway in this study. The mass/spring TMD is compact enough to be concealed within the walkway perimeter skin and so need not be visible. The pendulum, when tuned to 0.56 Hz, is 2.6- to 3.9-ft long and must therefore be exposed to view. Hence, as people walk across the walkway, the pendulum will visibly swing back and forth. An exposed structural element such as the pendulum TMD may actually enhance the space for structures like museums and educational facilities.

Wakway Response with a Tuned-Mass Damper

The MDOF dynamics model is extended to include the TMD degree of freedom, $q_T(t)$. The TMD tuned to the lateral resonance frequency of 0.56 Hz creates an antiresonance at that frequency where the response is much lower than without the TMD. The FRFs shown in Figure 8 illustrate the effect that a 1000-lbf pendulum has on the walkway. The ratio of the responses at 0.56 Hz is a critical measure of the TMD's performance. In the case of the pendulum TMD, the lateral acceleration response of the walkway is 36 times lower with TMD.

The mass/spring TMD is more effective than the pendulum. A 1000-lbf weight mounted between and supported by the two W-sections provides a 50-times reduction in the lateral acceleration.

The lateral acceleration felt by someone near midspan is plotted in Figure 9 and should be compared to the response with no TMD (Figure 6). The resonant build-up is eliminated because the original resonance at 0.56 Hz is replaced with an antiresonance. The dominant disturbing motion near 0.56 Hz is reduced to such an extent that it is not discernable in the plot.

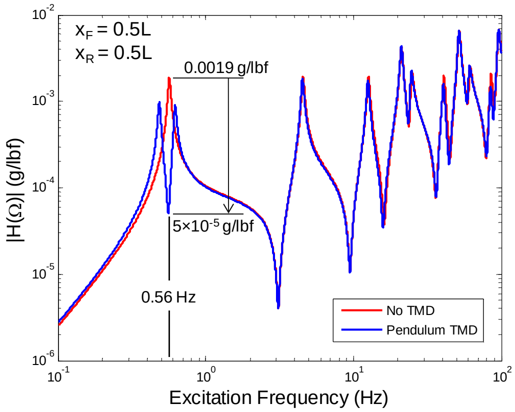

Figure 8 Comparison of FRFs With and Without the Pendulum TMD

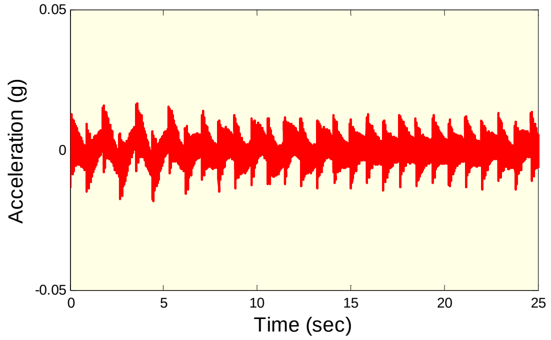

Figure 9 Lateral Acceleration at Midspan with a 1000-lbf Pendulum TMD

The peak acceleration magnitude is about 0.01g, which is only 1/4 of the original peak acceleration rather than the 1/36 reduction discussed above. Recall, the response shown in Figure 9 is the net motion across the full frequency spectrum, whereas the 1/36 reduction factor applies only in the narrow band around 0.56 Hz. People tend to be sensitive to motion at lower frequencies; motion above, say, 10 Hz causes little discomfort. The TMD acts as a narrow-band filter on the original motion by eliminating the most disturbing contribution to the motion. An illustration of the walkway cross-section with the pendulum TMD and a side-by-side comparison of the lateral displacements is shown in Figure 10.

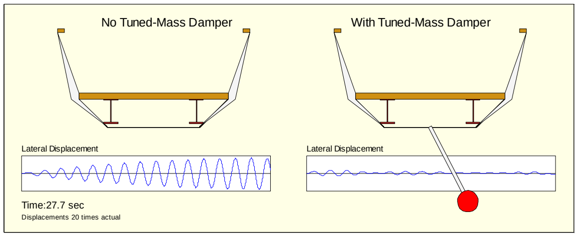

Figure 10 Cross-Section of Walkway with the Pendulum TMD and Displacement Comparison

CONCLUSIONS

Two distinct tuned-mass dampers are investigated here to control the lateral acceleration felt by pedestrians on a flexible mezzanine walkway. A MDOF dynamics model of the walkway is developed and used to determine the walkway resonance frequencies and mode shapes. The walkway in question has a fundamental lateral flexural mode at 0.56 Hz, which coincides with the fundamental lateral excitation frequency of walking-induced forces. Side-to-side acceleration levels approaching 0.2g at low frequencies is sufficient to cause people to feel as if they are on the verge of losing their balance. Five to ten people walking together on the subject walkway are sufficient to produce this level of acceleration.

The walkway dynamics model is extended to include two different forms of TMDs: a traditional mass/spring TMD and a pendulum TMD. A 1000-lbf TMD weight at midspan has a minor impact on the original structure, but reduces the lateral acceleration magnitude in the critical low frequency range by a factor of 36 to 50 with the pendulum or lumped mass/spring TMD, respectively.

The pendulum TMD is less effective than the more compact mass/spring TMD, but may add visual interest to the structure and surrounding space when the functioning TMD can be seen. The more compact mass/spring TMD has the advantages that it is more effective for a given added weight and can be easily concealed within the walkway's envelope.

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